A Coalition Analysis Algorithm with Application to the Zimbabwe Conflict

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Abstract-An algorithm is developed for considering coalitions when formally analyzing a given conflict. The new methodology is employed for studying the 1979 Lancaster Peace Talks concerning the civil war in Zimbabwe. Following an historical description of the Zimbabwe controversy, the conflict is analyzed using an improved metagame stability analysis algorithm. A comprehensive procedure is then presented for incorporating coalition considerations into the stability analysis of the conflict. Furthermore, a metric is introduced to determine when coalitions are most likely to form. The results of the study are in agreement with the actual historical events.

I. INTRODUCTION

NO MATTER what type of conflict is being studied, it is often advantageous to entertain coalitions in order to enhance one's understanding of the dispute. For instance, when analyzing a potential military confrontation, the possible resolutions can be greatly affected by coalitions forming between various players or participants. In an environmental dispute, the formation of an effective coalition may succeed in blocking a project that could adversely affect the environment. Consequently, by considering each of the meaningful coalitions that could arise in a particular conflict, a sensitivity analysis is employed to ascertain how various coalitions can affect the possible resolutions.

The purpose of this paper is to present a comprehensive procedure for studying coalitions that could form in any kind of conflict or game. The efficacy of the technique is demonstrated by a detailed analysis of the 1979 Lancaster Peace Talks which took place in London, England, for the purpose of finding a negotiated settlement to the civil war in Zimbabwe.

The systematic study of a conflict such as the Zimbabwe dispute involves two main steps [1]. First, the conflict is modeled by identifying the players or participants, the options or courses of action available to each player, and the players' preferences among the possible feasible outcomes. To model the conflict, a thorough understanding of the dispute is necessary. This can be gained by referring to the available published literature and, if possible, contacting people directly or indirectly involved. Accordingly, the history of the Zimbabwe crisis is outlined in the next section, before the actual modeling of the conflict.

The second major stage in the procedure is to perform a stability analysis of the feasible outcomes in a game in order to predict the possible resolutions or equilibria in the dispute. Thus a stability analysis is performed to predict possible resolutions to the Zimbabwe conflict. The stability analysis technique employed is the improved metagame analysis algorithms of Fraser and Hipel [1], which is based upon the metagame theory of Howard [2].

It should be emphasized that the coalition technique developed in this paper is independent of the type of stability analysis algorithm which is actually utilized for ascertaining the equilibria in the game. In certain situations, a practitioner may wish to employ another stability analysis algorithm such as the traditional metagame analysis method of Howard [2]. When the nonmyopic stability analysis technique of Brains and Wittman [3] is extended for use with games having more than two players, the reader may also wish to contemplate using the procedure. However, as demonstrated by a wide range of real-world applications, the improved metagame analysis algorithm [I] works well in practice, and consequently this method is used for analyzing the Zimbabwe dispute. Previously, the method of Fraser and Hipel [11 has been utilized for studying environmental [1], [4], energy [1], [5], labor-management [6], [7], military [1], [8]-[10], and political [I1] disputes. However, none of the foregoing applications dealt with coalitions, and in addition, a flexible procedure was not available for easily incorporating coalitions into the stability analysis algorithm of Fraser and Hipel [1].

In a conflict analysis study, the feasible outcomes are ordinally ranked from most to least preferred for each player. Such an ordering of outcomes is referred to as a player's preference vector. A detailed procedure is presented for combining the preference vectors for the two or more players in a specified coalition into an overall single coalition preference vector. The underlying principle for determining the coalition preference vector is that an outcome is not preferred by the coalition to another outcome if any member of the coalition does not prefer it. Equivalently, in order for an outcome to be preferred to another particular outcome by the coalition, all members of the coalition must prefer it.

II. HISTORICAL DESCRIPTION

Zimbabwe is a relatively prosperous country located in the interior of southern Africa. It has a total area of 391 000 km-about three times the size of England. Of the almost seven million people who live in Zimbabwe, the vast majority, approximately 95 percent, are black Africans,

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while the rest are mainly of European descent. Throughout its history, the country has been known by a number of names including Southern Rhodesia (1889-1965), Rhodesia (1965-1979), Zimbabwe-Rhodesia (1979-1980), and Zimbabwe (March 1980 to the present).

In spite of its prosperity, based on an economy of agriculture, mining, and manufacturing, Zimbabwe has recently had a turbulent history. Black nationalist groups, under patriotic front leaders Mugabe and Nkomo, had been fighting in earnest since the early 1970's to topple the white minority government headed by Rhodesian Front (RF) leader and Prime Minister Ian Smith. In addition, world opinion was against the whites. As a result, Rhodesia was not diplomatically recognized by many other countries, and international economic sanctions were brought against it. During the discussions held at Lancaster House in London from September to December of 1979, the country of Zimbabwe–Rhodesia (as it was known at the time) confronted the question of how to end a seven-year old civil war, remove economic sanctions, and gain international recognition as a majority rule government.

Zimbabwe's political troubles and, ironically, economic prosperity began in 1889 when Cecil Rhodes and other white settlers secured for the South African Company a royal charter for the area now known as Zimbabwe. Rhodes quickly subjugated the Matabele and Mashona people who were living there and in 1895 named the area Southern Rhodesia,

> . . . but it was not recognized by the British Government till the Southern Rhodesian Order in Council in 1898. [12]

In 1923, the white settlers voted for Southern Rhodesia to become a British colony. As part of the dominion of the United Kingdom, however, Britain insisted that the whites create a black majority rule government. The white minority obstinately resisted this British suggestion because

> . . . the settlers had voted to accept a constitution which save for a pretended and probably illusory reservation to the Imperial authority of control over native affairs, established local Responsible Government. [13]

In the late 1940's and early 1950's, movement towards a Central African Federation of Southern and Northern Rhodesia and Nyasaland was apparent because

> Economic arguments-the dollar-earning capacity of copper and tobacco-and strategic considerations-the danger of South Africa drawing territories north of Limpopo into her orbit... . [12]

On October 23, 1953, Britain allowed the formation of the soon-to-be-apparent white-dominated Federation. The Federation did not last, though, because of

> ... the alienation of African nationalists from the European electorate, which gathered momentum after 1958. By the end of 1962, all three Territorial Governments had ceased to support the continuation of the Federation. Succession had already been in principle conceded to Nyasaland and it was only a matter of time before the coalition government of Northern Rhodesia under Kaunda and Nkumbula would follow the same course... . [12]

The formal dissolution of the federation occurred on January 1, 1964.

In 1961, the whites tightened their control on the government of Southern Rhodesia by creating a single 65-member legislature in which only those of "superior means and education" (the whites and a few blacks) had the right to vote for 50 seats in parliament.

> By many, and not only Rhodesian Front (a political party consisting of a group of prominent white farmers headed by Ian Smith) supporters, it was claimed that the granting of the 1961 Constitution implied acceptance of Rhodesian independence if and when the Federation ceased to exist. [14]

Indeed, on November 11, 1965, a year after the demise of the Federation, the Southern Rhodesian people elected Ian Smith as their Prime Minister. Britain immediately imposed economic sanctions on Rhodesia and declared the government illegal. Not until December 1966 did Ian Smith and British Prime Minister Harold Wilson finally meet on the ship H.M.S. *Tiger* in the Mediterranian off Gibraltor for talks on the normalization of relations between the two countries. Wilson presented Smith with a lenient "Five-Principle" proposal which essentially allowed Rhodesia a British-recognized independence for a promise of eventual majority rule in the country. Smith refused these proposals, and as a result the United Nations Security Council imposed international economic sanctions on the country 15].

For the next five years, Smith's regime became more and more entrenched in its minority rule. The October 1968 "H.M.S. *Fearless*" talks, where British terms were similar to those given to Rhodesia in 1966, failed due to Smith's intransigence [16]. In June 1969, the Rhodesian Front party held a referendum restricted to whites only which overwhelmingly approved a Rhodesian constitution based on the 1961 revisions to the government [161. In March 1971, the Rhodesian Front swept all 50 seats in the legislature. In spite of growing criticism (13 bordering or near-bordering African states issued the Lusaka Manifesto denouncing the Rhodesian Front's 1969 referendum), guerrilla activity remained sporadic, and the United Nations sanctions proved ineffectual because of merchants operating out of countries violating the international sanctions.

By late 1971, however, the ailing Rhodesian economy, hurt by a serious drop in mineral prices and a poor foreign exchange, forced some white Rhodesian concessions. In November of 1971, Britain and Rhodesia reached an agreement based on a form of the Five-Principles [16]. However, the Pearce Commission, created by the British government to test the opinion of all segments of the Rhodesian population, found the agreement unacceptable to the majority. The black Rhodesian population wanted actual majority rule and not merely the promise of it as set out in the Five-Principles. As a result of the Pearce Commission's findings, international sanctions were retained and recognition withheld from Rhodesia.

Military, political, and economic pressures against Rhodesia intensified between 1972 and 1978. December

1972, when a white Rhodesian farm was subjected to rocket fire by guerrillas, is generally considered the start of the Rhodesian civil war [16]. In late 1974, the Portuguese surrendered a once impartial Mozambique to black nationalists [16]. By 1976, President Samora Marcel of Mozambique closed his borders and communications with Rhodesia in a show of displeasure with Rhodesian racist policies [16]. A marked decline in the white population due to emigration left a shortage of men in agriculture and industry during this period. Also, the oil embargo significantly affected the fuel hungry industry of Rhodesia.

During the early 1970's, many black opposition parties emerged.

> On December 8, 1974, an agreement was concluded at Lusaka, Zambia, by Bishop Abel Muzorewa of the African National Council (ANC), Joshua Nkomo of the Zimbabwe African People's Union (ZAPU), Ndabaningi Sithole of the Zimbabwe African National Union (ZANU), and James Chikerema of the Front for the Liberation of Zimbabwe (Frolizi), whereby the latter three would join an enlarged ANC executive under Bishop Muzorewa. On September 1976, Sithole announced that ZANU had withdrawn from the ANC, which, since its formation in December 1974, had been split into two wings led by Bishop Muzorewa and ZAPU leader Nkomo. Collaterally, Mugabe claimed the leadership of ZANU and the Sithole group within Rhodesia became known as ANC-Sithole, while the Muzorewa group became known as the United African National Council (UANC). (Furthermore), Mugabe... announced the formation of a Patriotic Front (PF) linking ZANU and ZAPU military units. [17]

Thus the black opposition can be broken down into the moderate black African parties UANC, ANC-Sithole, and ZUPO led by Muzorewa, Sithole, and Chirau, respectively, and overtly insurgent black African groups ZAPU and ZANU united under the PF lead by Nkomo and Mugabe, respectively.

Reacting to the intensified pressures, Smith held secret talks with a moderate black leader, Bishop Muzorewa, throughout 1973 and early 1974 [16]. These discussions proved to be futile. However, due to political pressure from not only Zambian President Kaunda but also the Rhodesian ally, South African Prime Minister Vorster, Ian Smith and patriotic front leader, Nkomo, met on August 25, 1975. Unfortunately, by March 1976, these talks had also broken down [16].

In March of 1978 "inside talks" between Ian Smith and the three moderate black African leaders (Muzorewa, Sithole, and Chirau) concluded the so-called Internal Settlement (IS). At the same time, "outside talks" were conducted between British and American emissaries, leaders of the PF (Nkomo and Mugabe), and the "Front Line Presidents" of Tanzania, Zambia, Mozambique, Botswana, and Angola. These outside talks concluded that the IS did not command sufficient authority to bring the war or international sanctions to an end [16].

In spite of world opinion, Zimbabwe-Rhodesia was created out of the IS on May 31, 1979. This ended 88 years of white domination, but a true majority-ruled independent government had not yet emerged. In essence, the 65-man Rhodesian legislature was increased to 100 with only 28 seats reserved for the whites; however, special privileges and powers were still retained by the whites. Muzorewa, after a campaign viewed by many blacks with extreme skepticism, gained a majority in the new government; Ian Smith became a cabinet minister without portfolio [16].

White Rhodesian hopes for recognition of their IS settlement faded quickly in 1979. The Zimbabwe-Rhodesian army now faced about 40 000 guerrillas, 13 000 of whom were thought to operate within the country (double the previous year). Throughout early 1979, Muzorewa shuttled between the African United Organization (an organization representing many African countries), the U.S., and Britain to gain international recognition for his government, but he was unsuccessful [181.

A hopeful breakthrough for the Zimbabwe conflict appeared on August 5, 1979, at the Commonwealth's Twenty-Second Conference in Lusaka, where new proposals were approved calling for a ceasefire, a new constitution, and an election supervised by the British government. Continuing the impetus, Britain formally invited the representatives of Muzorewa's government and the PF to Lancaster House, London. Finally, on September 11, 1979, with British Foreign Secretary Lord Carrington presiding, the Zimbabwe-Rhodesians and guerrillas met [18]. The three participants sought a way to end the fighting, remove adverse economic sanctions, and achieve international recognition for Zimbabwe-Rhodesia.

111. CONFLICT ANALYSIS

As mentioned in the introduction, a conflict study is executed by following two major stages. First, the historical information is systematically organized according to a formal structure by ascertaining the players, options, and each player's preferences among the possible feasible outcomes in the game. After this modeling process, the second stage is to perform a stability analysis in order to predict the possible resolutions or equilibria to the conflict. Each outcome is analyzed for stability for each player, and those outcomes which possess some type of stability for all the players form the set of equilibria. The foregoing conflict analysis procedures are now applied to the Zimbabwe dispute. However, the purpose of the paper is to give the detailed results for the new coalition analysis method, and consequently, readers desiring more extensive coverage regarding modeling and stability analysis may wish to refer to [1], [19].

Modeling the Conflict

The conflict, now in the form of an historical description, is formally modeled so that the stability analysis can be carried out at the next stage. To start, the Zimbabwe conflict is considered at only one point in time, September 11, 1979, the date the talks began at Lancaster House. Players or participants in the conflict that can take actions

TABLE I
FEASIBLE OUTCOMES IN THE ZIMBABWE CONFLICT

Rhodesians																				
Advocate IS	0	1	0	1	0	0	1	0	1	0	0	1	J	1	0	0	1	0	1	0
Escalate War	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0
Compromise	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1
PF																				
Accept IS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Escalate War	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0
Compromise	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1
British																				
Accept IS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Support PF	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
Compromise	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	.1
Decimalized Outcomes	144	145	146	147	148	160	161	162	163	164	272	273	274	275	276	288	269	290	291	292

which have significance to the other players are identified. As shown in Table I, the three players in the Zimbabwe conflict are the Rhodesians, the Patriotic Front (PF), and the British. The Rhodesian player, or actually the Zimbabwe-Rhodesian, in the terminology of September 1979, in the conflict represents not only Ian Smith's Rhodesian Front and the white minority in Zimbabwe-Rhodesia but also the moderate black leaders (Muzorewa, Sithole, and Chirau) and their following. The PF is composed of the Patriotic Front guerrilla group, headed by Mugabe and Nkomo and influenced by the Front Line presidents. The British includes only Britain which is influenced by the U.S.

Options or possible actions that can be taken by each player are found next. As can be seen in Table I, the Rhodesian's options would be to try to convince the PF and Britain to accept the Internal Settlement (Advocate IS), escalate the War (Escalate War), and if these two options are not possible, entertain serious conciliatory discussion with the PF on black majority rule (Compromise). The PF desire to gain control of Zimbabwe-Rhodesia, and thus escalating the war or compromise would be their two most preferable options. The remote possibility also exists that, due to pressure from the Front Line presidents, the PF may choose to accept the IS as another option. The British player's main objective is to convince the Rhodesians and PF to settle the dispute peacefully. The British options, therefore, are to support the Rhodesians (Accept IS), support the PF (Support PF), or act unbiasedly (Compromise). Each of the players followed by their options for the Zimbabwe conflict are listed on the left-hand side of Table I.

A possible selection of options by one particular player is referred to as a strategy, and the situation where each player chooses a strategy is called an outcome. An outcome is denoted in Table I by a column of ones and zeros, where

TABLE II
OUTCOMES REMOVED FROM THE ZIMBABWE CONFLICT

Rhodesian														
Advocate IS	-	-	-	-	-	-	-	1	-	-	-	-	-	-
Escalate War	-	-	-	-	-	-	1	-	-	-	-	-	-	-
Compromise	-	-	-	-	-	-	1	1	-	-	-	-	1	-
PF														
Accept IS	-	-	-	-	1	1	-	-	-	0	1	0	1	1
Escalate War	-	-	-	1	-	1	-	-	-	-	-	0	-	-
Compromise	-	-	-	1	1	-	-	-	-	-	-	0	-	-
British														
Accept IS	-	1	1	-	-	-	-	-	0	1	0	-	-	-
Support PF	1	-	1	-	-	-	-	-	0	-	-	-	-	-
Compromise	1	1	-	-	-	-	-	-	0	-	-	-	-	-

"1" is placed opposite an option to indicate that it is selected by the player controlling the option and a "0" means that the option is not chosen. Thus the first outcome from the left in Table I represents the situation where the PF takes the strategy of escalating the war against the Rhodesians and the British take the strategy of supporting the PF. In the text, an outcome is written horizontally, so that (000 010 010) is the representation of the first outcome in Table 1.

For a game or conflict with n options, the total number of possible outcomes would be 2". In the Zimbabwe conflict, 2^9 or 512 possible outcomes exist. However, some outcomes cannot or are not likely to occur and therefore can be removed from the total set of possible outcomes using techniques which can be calculated by hand [191 or using a microcomputer [20], [21]. Each column in Table II represents a set of outcomes that can be removed. A dash

TABLE III
DECIMALIZED PREFERENCE VECTORS FOR ZIMBABWE DISPUTE

Preference Vector Position	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Rhodesian preference vector	292	288	289	290	291	160	161	164	162	163	272	273	276	274	275	144	145	148	146	147
PF's preference vector	$\overline{292}$	$\overline{164}$	148	276	160	161	288	289	163	162	291	290	144	145	147	146	272	273	275	274
British preference vector	292	164	276	148	288	289	160	161	291	290	163	162	272	273	275	274	144	145	147	146

means that the entry can be either one or zero. For example, the first column on the left in Table II represents 2^7 or 128 impossible outcomes because of the seven dashes it contains. The actual interpretation of (--- --- -11) is thus: no matter what other options are selected, an outcome is infeasible if the British support the PF and are, at the same time, impartial at the Lancaster talks. The total number of removable outcomes contained in all the columns in Table II is 492, and when these are removed, the 20 feasible outcomes appearing in Table I remain.

Following the isolation of the 20 feasible outcomes, a preference vector can be formed for each player by ranking the outcomes in descending order from the most preferred outcome on the left to the least preferred on the right. The outcomes in these preference vectors can be expressed more efficiently by using the process of decimalization where one decimal symbol, instead of an entire column of ones and zeros, is used to represent a particular outcome. This process considers each element in the outcome as a binary number, where the entry in the first position corresponds to 2^0, the entry in the second position refers to 2^1, and so on. These binary numbers can be converted to decimal form to make them more compact by adding together the power-of-two values. For example, the outcome (000 010 010) has the result

$$(0 \times 2^0) + (0 \times 2^1) + (0 \times 2^2) + (0 \times 2^3) + (1 \times 2^4) + (0 \times 2^5) + (0 \times 2^6) + (1 \times 2^7) + (0 \times 2^8) = 144,$$

as shown in the first column of Table I.

The decimalized preference vectors for each of the players in the Zimbabwe conflict are shown in Table III. These vectors are ordered from most preferred on the left to least preferred on the right. For example, the Rhodesian preference vector can be broken into four main blocks, each containing five outcomes. The four main blocks from most to least preferred for the Rhodesians are as follows: the PF and British compromise outcomes (292, 288, 289, 290, and 291), the PF compromise with British support (160, 161, 164, 162, and 163), the PF escalates the fighting at the same time the British are trying to act unbiasedly in the talks (272, 273, 276, 274, and 275), and, least preferred, the PF escalates the civil war with British support (144, 145, 148, 146, and 147). Similarly, the PF's preference vector may be subdivided into three main blocks, from most to least preferred: the Rhodesians compromise (292, 164, 148, and 276), the PF compromises (outcomes 160–144), and lastly, the PF escalates the war (outcomes 145–274). Finally, the British player has three sets of outcomes which are quite similar to those of the PF's preference vector. In fact, the only consistent difference between the two preference vectors is that within each of the outcome sets, the British prefer the situation of mediating the talks impartially as opposed to politically supporting the PF. (Naturally, the PF would prefer the reverse of this.) For example, the outcome pair (148, 276) in the PF preference vector appears as (276, 148) in the British preference vector in Table III.

In a conflict study, some outcomes can be judged to be equally preferred, and this is indicated by a bridge placed across the equally preferred outcomes. Neither the Rhodesian nor British players have equally preferred outcomes; however, outcomes 292 and 164 for the PF in Table III are considered equally preferred.

When a player has the ability to unilaterally change to a new outcome, he is said to have a unilateral improvement (UI), from the original outcome to the new outcome. Consider outcome (000 010 001) or 272 and (100 010 001) or 273 with respect to the Rhodesian player in Table I. Specifically, notice that the Rhodesians can unilaterally improve their position by moving from their strategy (100) in outcome 273 to strategy (000) in outcome 272, where the strategies of the other two players remain fixed at (010 001). This is an example of a Rhodesian UI. A UI is indicated in Table IV by writing it below the outcomes in the preference vector from which the player has the UI. Consequently, the Rhodesian player's UI from 273 to 272 is indicated by writing 272 below 273. A player may have more than one UI from an outcome. In this case, they are written in a column under the outcome from the most preferred UI at the top to the least preferred UI at the bottom. For instance, outcome 276 in the Rhodesian preference vector has two UI's: 272 and, less preferred, 273.

Similar to the unilateral improvement (UI) is the unilateral change (UC). When a player has the ability to change an outcome by unilaterally changing to a new

TABLE IV
STABILITY ANALYSIS OF THE ZIMBABWE CONFLICT WITHOUT A COALITION

Rhodesian Preference Vector

E	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	Overall stability
r	u	u	u	u	r	u	u	u	u	r	u	u	u	u	r	u	u	u	u	Rhodesian stability
292	288	289	290	291	160	161	164	162	163	272	273	276	274	275	144	145	148	146	147	Rhodesian preference vector
	292	292	292	292		160	160	160	160		272	272	272	272		144	144	144	144	Most preferred UI's
		288	288	288			161	161	161			273	273	273			145	145	145	Next most preferred UI's
			289	289				164	164				276	276				148	148	Third most preferred UI's
				290					162					274					146	Least preferred UI's

PF's Preference Vector

r	r	s	N	r	r	r	r	r	r	r	r	u	u	u	u	u	u	u	u	PF stability
292	164	148	276	160	161	288	289	163	162	291	290	144	145	147	146	272	273	275	274	PF preference vector
		164	292									160	161	163	162	288	289	291	290	UI's

British Preference Vector

r	a	r	s	r	r	u	u	r	r	u	u	r	r	r	r	u	u	u	u	British stability
292	164	276	148	288	289	160	161	291	290	163	162	272	273	275	274	144	145	147	146	British preference vector
	292		276			288	289			291	290					272	273	275	274	UI's

outcome where both outcomes involved are equally preferred, he is said to have made a UC from the original outcome to the new outcome. An example of a UC does not exist in Table IV; however, if outcomes 272 and 273 from the previous example had been equally preferred, then movement between these two outcomes would have been considered a UC by the Rhodesian player. Movement is assumed to be two-way between outcomes, and thus not only would the UC 273 appear beneath outcome 272 but also the UC 272 would appear beneath outcome 273. Like UI's, a player may have more than one UC from an outcome, but unlike UI's the UC's need not be written in any order beneath the outcome from which the UC occurs. Collectively, UI's and UC's are known as " unilateral movements."

Stability Analysis

Each of the outcomes listed in the preference vector in Table IV is analyzed for stability for each player separately and also across the players. An outcome is stable for an individual player if it is not reasonable for him to move from the outcome by switching his strategy. An outcome has overall stability and is called an equilibrium if it is stable for all players and hence constitutes a possible solution to the conflict [1], [19].

An outcome can be stable for a particular player for a number of reasons. A simple form of stability occurs when an outcome does not have a UL This type of stability is referred to as being "rational" and appears in the stability tableau as an "r" over the particular outcome. A rational outcome for a player does not have a Ul because the player cannot improve his strategy without affecting the other players. For example, the Rhodesian player cannot improve from his strategy of (000) in (000 010 001) or 272 without altering the (010 001) portion of 272 and contradicting the definition of a UL Thus 272 for the Rhodesian player is rational and is shown in Table IV as not having any UI's. Note that, in contrast to an outcome with a UI, an outcome with one or more UC's but no Ul's is rational. A player will not, under any circumstances, move from one outcome to another if both outcomes are equally preferred to one another. He does not move because he will not improve in his preference position by moving between equally preferred outcomes.

More complicated forms of stability and instability occur for the UI's. If for all UI's available to an outcome for a particular player, credible actions can be taken by the other player(s) which result in a less or equally preferred outcome for the particular player, the outcome is "sanctioned" and labeled "s." The possibility that a worse or equally preferred outcome could result from changing his strategy deters the player from moving away from the outcome under consideration. For example, in Table IV, although the PF player has a Ul from 148 to 164, the Rhodesians have a Ul from 164 to 160, where 160 is less preferred by the PF player than 148, and thus outcome 148 for the PF player is sanctioned. Instability occurs when the player has at least one Ul from which the other player(s) can take no credible action which would result in a less or equally preferred outcome than the one from which the player is improving. An " unstable" outcome is marked as "u." For instance, in Table IV outcome 273 for the Rhodesian player is unstable because the other players cannot stop the Rhodesians from unilaterally improving to outcome 272.

UC's, in addition to UI's, may be used for sanctioning purposes. Modifying the example just given, the Rhodesians could have had a UC rather than UI from 164 to 160 if 164 and 160 had both been located beneath the same equally preferred bridge. This UC, just as the UI previously, would have sanctioned 148 for the PF player. Thus an outcome with a UC must be stable, and a UC can only be used for sanctioning purposes.

If an outcome is unstable for two or more players, a check is made for stability by simultaneity. This check

TABLE V
COALITION ANALYSIS BETWEEN PF AND BRITISH

Preference Vector Position	1	2	3	4	5'	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
Individual Preference Vectors																					
First Above: PF	292	164	148	276	160	161	288	289	163	162	291	290	144	145	147	146	272	273	275	274	
Baseline: British	292	164	276	148	288	289	160	161	291	290	163	162	272	273	275	274	144	145	147	146	
Part 1. Individual Bridge Removal																					
First Above: PF	292	164	148	276	160	161	288	289	163	162	291	290	144	145	147	146	272	273	275	274	
Baseline: British	292	164	276	148	288	289	160	161	291	290	163	162	272	273	275	274	144	145	147	146	
Part 2: Coalition Bridge Creation																					
PF/British Coalition	292	164	276	148	288	289	160	161	291	290	163	162	272	273	275	274	144	145	147	146	
Part 3. Unilateral Movements (UI' and UC's)																					
PF/British Coalition	292	164	276	148	288	289	160	161	291	290	163	162	272	273	275	274	144	145	147	146	
		292	292	292									288	289	291	290	288	289	291	290	UI's
			164	164									160	161	163	162	160	161	163	162	
			148	276	160	161	288	289	163	162	291	290	144	145	147	146	272	273	275	274	UC's

region, are designated by drawing a line across the top of the affected outcomes. The reason for this will be given later.) Thus a degree of "order" or agreement between coalition members about the preference ordering of some outcomes exists in nonordinal regions. Those nonordinal outcomes which have the same ordering for all coalition members are referred to as ordered nonordinal outcomes, and those which have conflicting ordering between coalition members are known as unordered nonordinal outcomes.

Equally preferred outcomes are considered to be a special case of unordered nonordinal outcomes. Whereas the general nonordinal outcomes may be unordered with respect to only some of the other outcomes in the nonordinal region, the equally preferred outcomes are considered unordered with respect to all other outcomes in an enclosed region. For example, outcome 288 is unordered with respect to outcomes 160 and 161 but ordered with respect to outcome 289 in the (288, 289, 160, 161) nonordinal region given in Part 2 of Table V. If these four outcomes were to appear in an equally preferred region, they would all be unordered with respect to one another. Since equally preferred outcomes are a special case of nonordinal outcomes, both are notationally indicated in the same manner in coalition analysis by a connecting bridge placed above the appropriate outcomes. Thus, although outcomes (292, 164) are equally preferred before the coalition process and outcomes (276, 148) are nonordinal after the coalition in Table V, a single line or bridge is drawn across both these pairs of outcomes to indicate their nonordinality. The order which can be found in nonordinal regions is used in an analysis of coalition preference vectors, 'as presented later in this paper.

A couple of questions arise concerning the ordinal coalition analysis. First, by how much one player prefers one outcome over another is not known. If the British strongly desired 276 to 148 and the PF only mildly wanted 148 over 276, then perhaps the outcomes should be ordered in favor, of the British and not be designated as nonordinal in the coalition preference vector. The implicit assumption made is that, although differing strengths of preferences may exist for one outcome over another between partners in a coalition preference vector, the cold hard truth of disagreement remains. Whether the partners disagree on ordering by a lot or by a little, the fact of disagreement inevitably leads to these outcomes being designated nonordinal. Thus outcomes 276 and 148 for the PF/British coalition are nonordinal.

A second question arises concerning the insensitivity of the analysis to the relative strengths between players. For instance, if the PF were the stronger member of the coalition supporting outcome 148 over 276 and the British favored outcome 276 to outcome 148, the possibility exists that the PF's would retain their ordering of outcomes in the coalition preference vector. However, it is reasoned that the strengths of other players is taken into account in the ordering of an individual's preference vector. For example, the British ordered their 276 and 148 outcomes by taking into account the strength of the PF player. That is, the British could have positioned (276, 148) not at position (3, 4) in the individual preference vector part of Table V but further down their preference vector, say at (11, 12). However, because the British figure they are strong enough relative to the PF, they have located the (276, 148) outcomes at their more preferred positions.

The method presented in the next three subsections is used to create a coalition preference vector suitable for use in stability analysis. This coalition preference vector may be derived from any number of individual coalition preference vectors which are equal in length. The first section

describes the implementation of a coalition algorithm to identify the nonordinal outcomes in the coalition preference vector and to determine the appropriate UI's and UC's for the coalition preference vector. In the second section, a stability analysis of the game containing a coalition is performed and compared to the stability analysis of the game without any coalitions. The last section presents a possible metric for indicating the compatibility of coalescing members.

COALITION ALGORITHM

The coalition algorithm is a systematic method to compare the ordering of every combination of outcome pairs between two or more players' preference vectors which are equal in length in order to obtain a single coalition preference vector. Although a rather lengthy description is required to properly explain how the algorithm works, one familiar with it would find the process to be simple and quick to perform. The coalition preference vector is created from one of the merging players' preference vectors which is stripped of its equally preferred bridges and referred to as the baseline. The ordering of the coalition baseline preference vector is founded upon designating each contained outcome pair as ordinal or nonordinal. Ordinal outcomes are those outcomes which are not found beneath a nonordinal bridge. The baseline outcome pair is compared to another "Nth-above" player's preference vector. In Table V the British player's preference vector is the baseline, and the PF's preference vector is the first-above preference vector. (If a third player were to be merged with the British and PF, he would be designated the second-above vector.)

The coalition algorithm contains three parts. The first section involves removing as best as possible the nonordinal bridges from the individual preference vectors. Removal of these bridges simplifies the second part of the algorithm, which is to determine the nonordinal bridges for the coalition preference vector. The third component involves determining the unilateral movements for the coalition preference vector.

Part 1 of Coalition Algorithm-Individual Bridge Removal: **The first part of the algorithm involves removing as many bridges as possible from the preference vectors of coalescing players. In essence, a bridge is removed if the outcomes in this area are able to be moved in such a way within one player's preference vector to permit matching the identical outcome ordering in another player's preference vectors. The bridged player is able to move because he treats the affected outcomes with indifference and thus will order them according to another's ordering. The other player can find his preference ordering with respect to the first's ordering in one of three situations. Either the second player's outcomes may be ordinal and positioned under the first's outcomes, or the second's outcomes may be equally preferred and fall beneath the first's bridged outcomes, or lastly, the second's outcomes may simply not fall under the first's outcomes. These three cases are described in greater detail as follows.**

TABLE VI
PART 1 OF COALITION ANALYSIS ALGORITHM: INDIVIDUAL BRIDGE REMOVAL EXAMPLES

Case 1. Equally Preferred-Ordinal Example (Hypothetical)												
position	1	2	...				position	1	2	...		
PF	164	292				Becomes	PF	292	164	...		
British	292	164					British	292	164	...		
British Baseline	292	164	...				British Baseline	292	164	...		
Case 2. Equally Preferred-Equally Preferred Example (Hypothetical)												
position	1	2	••				position	1	2	...		
PF	164	292	...			Becomes	PF	292	164	...		
British	292	164	...				British	292	164	...		
British Baseline	292	164	...				PF/British Coalition	292	164	...		
Case 3. No Match-Up Example (Hypothetical)												
position	1	2	•••	6	7		position	1	2	...	6	7
PF			..	292	164		PF			...	292	164
British	292	164	•••			Becomes	British	292	164	...		
British Baseline	292	164	...				British Baseline	292	164	...		

Equally preferred-ordinal case: **An equally preferred ordinal case arises when an equally preferred outcome in an individual preference vector is in a position which can be moved to the position of the identical (but ordinal) outcome of another player's preference vector. For example, consider outcomes 292 and 164 in the PF individual preference vector of Table V. Both outcome 292 and 164 can be moved within the designated equally preferred region. Thus 292 and 164 could both be located either at preference vector positions (1, 2) or (2, 1). In this example, (292, 164) just happens to be positioned at (1, 2), and it therefore lines up with the position of the ordinal outcomes (292, 164) in the British's preference vector. This is a situation of the equally preferred-ordinal case. Indeed, if outcomes (292, 164) had been at positions (2, 1) rather than (1, 2) for the PF, this revised situation would also be an example of the equally preferred-ordinal case. Refer to Case 1, of Table VI for an illustration of this latter hypothetical example. In both cases, once the outcomes are reshuffled, all affected bridges are removed. Thus the bridge above (1, 2) in the PF preference vector is removed in Part 1 of the algorithm.**

Equally preferred-equally preferred case: **The second case, the equally preferred-equally preferred situation, arises where a pair of outcomes are equally preferred in both the baseline and Nth-above preference vectors. In this situation, the individual preference vector bridges are initially removed in part 1, but the pair of appropriate coalition preference vector outcomes are designated equally preferred in part 2. The affected outcomes are "solved" or, in other words, indicated as equally preferred in the coalition preference vector during the first part of the coalition algorithm and ignored in the second part of the algorithm. For example, consider the hypothetical example in Case 2 of Table VI, where outcomes 292 and 164 are imagined equally preferred for both the PF and British. In this case, the bridges over both these outcomes would be removed in the preference vectors in Part 1 of the coalition algorithm** and a bridge automatically placed over (292, 164) in the coalition preference vector of Part 2 of the algorithm.

No-matchup case: The third case arises where the equally preferred outcomes are not able to match up with appropriate ordinal outcomes. In this situation, the equally preferred bridges are removed, leaving the revealed outcomes in their unordered positions. This case occurs when the position of the equally preferred region, of which a particular outcome is a part, in one preference vector does not cover an equivalent region that contains the same ordinal outcome in the merging preference vector. Since the particular outcome cannot move outside its equally preferred region and the equally preferred region cannot move to another position in the preference vector, a match in position between the particular equally preferred outcome and the ordinal outcome is not possible. When the equally preferred bridge is removed, the outcomes affected are left in unordered positions, and the number of strictly ordered outcomes remains the same. An example of this case does not appear for the PF/British coalition in the Zimbabwe dispute. However, consider the simple imaginary example given in Case 3 of Table VI, where the British have equally preferred outcomes (292, 164) at the (1, 2) positions, and the PF have their ordinal outcomes (292, 164) at positions (6, 7). In this instance, all outcomes would remain in their individual preference vector positions when the equally preferred bridge is removed from above the British (292, 164) outcomes in the first step of the algorithm.

This completes the description of the first half of the coalition algorithm. In summary, the three cases are as follows.

1) Equally preferred-ordinal outcome case: The equally preferred bridges are removed, and the affected outcomes in the individual preference vectors are designated ordinal.
2) Equally preferred-equally preferred outcome case: The equally preferred bridges are removed from the affected individual preference vector outcomes, and the appropriate outcomes in the coalition preference vector are designated equally preferred.
3) No match-up case:. The equally preferred bridges are removed, and the affected outcomes remain unchanged in position in the individual preference vectors.

Part 2 of Coalition Algorithm: Coalition Bridge Creation: The determination of the placement of the nonordinal bridges in the coalition preference vector is undertaken in the second part of the algorithm. The coalition vector is constructed in three steps: leaving the previously designated equally preferred outcomes from Case 2, Part 1 of the algorithm as is; regarding all individual preference vector outcomes whose order and position is exactly the same for each of the players (or "strictly ordered" as ordinal in the coalition preference vector); and designating all other outcomes as nonordinal.

In the Zimbabwe conflict, the final preference vector contains no previously designated equally preferred sections.

TABLE VII
PART 2 OF COALITION ANALYSIS ALGORITHM: COALITION BRIDGE CREATION EXAMPLES

Case 1. Coalition Equally Preferred (Continued from Case 2 - Part 1)											
position	1	2	...				position	1	2	...	
PF	292	164	...			Remains	PF	292	164	...	
British	292	164	...				British	292	164	...	
PF/British Coalition	292	164	...				PF/British Coalition	292	164	...	
Case 2. Individual Strictly Ordered (Taken from Part 2 of Table V)											
position	1	2					position	1	2	...	
PF	292	164				Remains	pp	292	164	...	
British	292	164					British	292	164	...	
British Baseline	292	164	...				PF/British Coalition	292	164	...	
Case 3. Non-ordinal Outcomes (Taken from Part 2 of Table V)											
position	5	6	7	8			position	5	6	7	8
PF	160	161	288	289		Becomes	PF	160	161	288	289
British	288	289	160	161		First:	British	288	289	160	161
British Baseline	288	289	160	161			PF/Brit coalition	288	289	160	161
							position	5	6	7	8
						Then	PF	160	161	288	289
						Finally:	British	288	289	160	161
							PF/Brit Coalition	288	289	160	161

However, continuing with the hypothetical example for Case 2, in Table VI where equally preferred bridges for the PF and British individual preference vectors have been removed and a bridge automatically placed over (292, 164) in the coalition preference, one finds, as shown in Case 1 of Table VII, that the bridge over (292, 164) in the PF/British coalition simply remains as is: untouched.

Only two sets of individual outcomes, 292 and 164, are strictly ordered in the Zimbabwe conflict. As can be seen both in Part 2 of Table V and highlighted in Case 2 of Table VII, these outcomes are located in the first and second positions of the PF/British coalition preference vector because they both are located in the first and second positions of the individual PF and British preference vectors.

The third situation arising in a merging process involving a "bridge expansion" technique is amply represented in the PF/British preference vector of the Zimbabwe conflict, since the rest of the outcomes in this coalition are nonordinal. Consider outcomes 276 and 148 in Part 2 of Table V. As indicated previously, the PF and British prefer these outcomes in an opposite manner to one another. More specifically, note that the location of the (148, 276) outcome pair for the PF is the (3, 4) position and for the British is the (4, 3) position. If one of these players were to interchange his outcome pair each by one position, outcome pair (276, 148) for both would become strictly ordered. In other words, (276, 148) is only slightly disordered for the PF/British coalition, and this is shown by a short nonordinal line extending across the top of only these two outcomes.

In general, the larger the distance between identical outcomes in different preference vectors, the larger the nonordinal area. Consider outcomes 288, 289, 160, and 161 shown in Case 3, Part 2 of Table VII (taken from Part 2 of

Table V). In particular, notice outcome 288. The nonordinal line must extend at least from outcome 288, the fifth position in the baseline preference vector, to outcome 160, the seventh position in the baseline preference vector, because outcome 288 occurs in the seventh position of the first-above preference vector. The PF/British partnership is unable to decide whether to position outcome 288 in the fifth or seventh positions of its coalition preference vector. This indecision in positioning is indicated by the nonordinal line drawn between the 288 and 160 outcomes in the coalition preference vector. The search for nonordinal outcomes does not end here as the question arises as to how outcome 289, inadvertently enclosed by the nonordinal line between outcomes 288 and 160, is to be classed in the coalition preference vector. Outcome 289 appears in the eighth position of the PF preference vector or the location of outcome 161 in the British preference vector. The disagreement between the PF and British about the positioning of outcome 289 causes the extension (thus the term bridge expansion technique) of the nonordinal line from outcome 288 to stretch down to the eighth position or outcome 161 in the coalition preference vector. In regard to outcome 160, the PF and British disagree once again about the position of this outcome. However, this time, the position of the 160 outcome (fifth and seventh positions for the PF and British, respectively) for both players falls within the previously developed nonordinal area (fifth, sixth, seventh, and eighth positions). Thus the nonordinal line remains the same length. Similarly, the coalescing partners disagree about the positioning of outcome 161 (sixth and eighth positions of the PF and British, respectively), but once again its position falls within the previous nonordinal area. At this point the nonordinal region stops growing to remain at four outcomes in length.

Once a nonordinal area has stopped expanding, a few things may happen: another nonordinal area can begin (as is the case for outcome 291, the next outcome after 161 as shown in Part 2 of Table V), an ordinal region may appear, or the end of the coalition preference vector may be reached. A coalition preference vector consisting of a mixture of ordinal and nonordinal outcomes emerges once the end of the coalition preference vector is reached.

Comparing and resolving two outcome pairs across more than two preference vectors is merely an extension of the above coalition technique. Instead of progressing through the baseline preference vector and looking above to only the first-above outcome, the search is extended to all Nth-above preference vectors. The preference vector which has the outcome furthest out of place with respect to the position of the baseline preference vector is chosen as the extent of the growing nonordinal area. With the exception of this modification, the coalition algorithms for more than two players are exactly the same for two players.

In summary, the steps involved in the second part of the coalition algorithm are as follows.

1) Previously designated equally preferred outcomes from Case 2, Part 1 of the algorithm are left as is.

TABLE VIII
PART 3 OF COALITION ANALYSIS ALGORITHM: UNILATERAL MOVES DETERMINATION EXAMPLES

Case 1. Unilateral Improvement (Taken from Part 3 of Table V)

position	1	2	3	4	..
PF/British	292	164	276	148	...
				292	(a UI because 292 is preferred to 148)

Case 2. Unilateral Change (Taken from Part 3 of Table V)

position	1	2	3	4	..
PF/British	292	164	276	148	•••
				148 2'76	(both UC's because 276 and 148 are equally preferred)

2) All strictly ordered individual outcomes in Part 1 of the coalition algorithm are designated ordinal in the Part 2 coalition preference vector.
3) All other outcomes are designated nonordinal according to the bridge expansion technique.

Part `3 of Coalition Algorithm: Unilateral Movement Determination: This section describes a simple method to determine the unilateral movements from the outcomes in any coalition preferencevector. Unilateral movement within coalition preference vectors is very similar to unilateral movement in individual preference vectors. In both situations, only two types of unilateral movements exist: unilateral improvement (UI) and unilateral change (UC). The analyst, though, must be careful to recognize that, when dealing with the unilateral movements of individual players, all other strategies except the one for the player making the movement remain fixed; in contrast, when dealing with coalitions, all other strategies except that of the merging (more than one) players making the movement remain fixed. Consider Part 3 of Table V. The PF/British coalition player in the lower portion of Table V can make a unilateral movement from outcome 148 or (001 010 010) to outcome 276 or (001 010 001) because strategy (001) of the player outside the coalition, the Rhodesians, remains the same for both these outcomes.

Coalition preference vectors, like individual preference vectors, have nonordinal outcomes which can have either UI's or UC's. The unilateral movements of nonordinal outcomes are designated either UI's or UC's according to whether the outcomes are ordered or unordered, respectively. Ordered nonordinal outcomes or outcomes having the same ordering for all coalition members treat their unilateral movements as UI's. Unordered nonordinal outcomes or outcomes having conflicting ordering among the coalition members have their unilateral movements treated as UC's. The Ul's and UC's have been divided into different lines below the coalition preference vector. For example, the unilateral movement 292 for nonordinal outcome 148, taken from Part 3 of Table V and highlighted in Case 1 of Table VIII, has been designated a UI because, for both the PF and British, outcome 292 is preferred to outcome 148. On the other hand, the unilateral movement 276 for nonordinal outcome 148, also taken from Part 3 of Table V and highlighted in Case 2 of Table VIII, has been designated a UC because of the reversed ordering of out-

TABLE IX
STABILITY ANALYSIS OF THE PF/BRITISH COALITION

Rhodesian Preference Vector

	X	X	X	X	E	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Er	u	u	u	u	r	u	u	u	u	r	s	s	u	u	r	u	u	u	
292	288	289	290	291	160	161	164	162	163	272	273	276	274	275	144	145	148	146	147
	292	292	292	292		160	160	160	160		272	272	272	272		144	144	144	144
		288	288	288			161	161	161			273	273	273			145	145	145
			289	289				164	164				276	276				148	148
				290					162					274					146

PF/British Coalition Preference Vector

r	d	u	u	r	r	r	r	r	r	r	r	u	u	u	u	u	u	u	u
292	164	148	276	288	289	160	161	291	290	163	162	272	273	275	274	144	145	147	146
	292	292	292									288	289	291	290	288	289	291	290
		164	164									160	161	163	162	160	161	163	162
		276	148	160	161	288	289	163	162	291	290	144	145	147	146	272	273	275	274

TABLE X
ORDINAL OUTCOME GROUP METRIC

Preference Vector Position	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Total Ordinal Outcome Groups
1. Individual Preference Vectors																					
Rhodesian Preference Vector	292	288	289	290	291	160	161	164	162	163	272	273	276	274	275	144	145	148	146	147	
PF Preference Vector	292	164	148	276	160	161	288	289	163	162	291	290	144	145	147	146	272	273	275	274	
British Preference Vector	292	164	276	148	288	289	160	161	291	290	163	162	272	273	275	274	144	145	147	147	
2. Coalition Preference Vectors																					
Rhodesian/PF Coalition	292	164	148	276	160	161	288	289	163	162	291	290	144	145	147	146	272	273	275	274	2
Rhodesian/British Coalition	292	164	276	148	288	289	160	161	291	290	163	162	272	273	275	274	144	145	147	146	3
PF/British Coalition	292	164	276	148	288	289	160	161	291	290	163	162	272	273	275	274	144	145	147	146	6

comes 276 and 148 in the PF and British preference vectors.

In summary, the steps involved in the third part of the coalition algorithm are as follows.

1) Unilateral movements for ordered nonordinal outcomes are UI's.
2) Unilateral movements for unordered nonordinal outcomes are UC's.

Stability Analysis for the Coalition Preference Vector

Once the coalition preference vector for the PF and the British are determined, along with UI's and UC's, another stability analysis can be performed as shown in Table IX. Indeed, comparing Tables IV and IX, the equilibrium results of the stability analysis before the PF and British merge can be seen to be different than the results of the stability analysis after they merge. In particular, a new equilibrium is.seen in the analysis of the PF/British coalition at outcome 160. Outcome 160 is the situation where the Rhodesians do nothing, the PF indicates a willingness to compromise, while the British support the PF. The consequence of this analysis is that the British, who were sympathetic to the goals of the PF, could not form an overt coalition with this player because by doing so the coalition would have to behave in such a manner as to result possibly in the less preferred stalemated situation of outcome 160. However, by acting independently, the British ensured that the resolution to the conflict was their favored equilibrium, outcome 292. Thus it was not necessary for the PF and British to coalesce, and this is what happened historically.

Compatibility of Coalition Members

This section describes the use of the number of ordinal outcome groups in the coalition as a metric indicating the compatibility between players. An ordinal outcome group is defined as either a single outcome which is not under a bridge or a group of outcomes joined by a bridge in a coalition preference vector.

The members in a coalition are assumed to be more compatible with one another the greater the amount of agreement they have over preference vector ordering. The greater the agreement over ordering, the greater the number of ordinal outcome groups which will appear in the coalition preference vector. Consider Table X where the three players' preference vectors for the Zimbabwe conflict appear, as well as the three possible coalitions between players. In the PF/British coalition preference vector, only two single ordinal outcomes, 292 and 164, actually exist. However, each block of nonordinal or equally preferred outcomes could be counted as one ordinal outcome group because each nonordinal block is preferred to the next nonordinal block. For example, the nonordinal block of outcomes 276 and 148 is preferred to the nonordinal block of outcomes 288, 289, 160, and 161. Thus the PF/British coalition preference vector has a total of six ordinal outcome groups. Similarly, the Rhodesians/British and Rhodesian/PF coalition preference vectors have three and two ordinal outcome groups, respectively.

Since the largest number of ordinal outcome groups of the three coalitions is six, the PF/British coalition, according to the metric, is the most compatible of the coalitions. The least likely merger occurs for the Rhodesian/PF coalition. These insights gained from the strictly ordered outcome group measure conform to historical events. The PF and British were more closely related to one another then either was to the Rhodesians, because both essentially wished majority rule in Zimbabwe. Their means to attain this end differed, however, as the PF were more ready to seek a military solution than were the British.

V. OTHER COALITION RESEARCH

Previous work in coalition analysis is reviewed in order to explain how it relates to the coalition research presented in this paper. First, the major difference between this and most other coalition analysis work is outlined. Second, two changes to coalition theory are compared to the work in this paper. Specifically, Brams' [22] "winning" concept of coalitions and Arrow's [23] "general possibility theorem" are discussed.

Ordinal Versus Cardinal Approaches

The single major distinction between the research in this paper and many other works in the coalition analysis field [24]-[33] is that the former is based on an ordinal approach whereas the latter are based on a cardinal approach. The ordinal approach is defined as the situation where players coalesce based on their knowledge of the relative ranking of the feasible outcomes. The cardinal approach is the case where players coalesce based on their knowledge of the distribution of weights representing such things as resources, votes, and power, and consequently, a real number or utility value can be assigned to each outcome.

Clearly, because the ordinal approach necessitates knowing only relative ordering, whereas the cardinal approach demands detailed data on the distribution of weights, the former approach requires less information than the latter in implementation. Since obtaining the detailed information required by the cardinal approach may often be unrealistic, the ordinal approach would be the only practical alternative to performing coalition analysis. Furthermore, the successful application of the ordinal technique to the Zimbabwe dispute demonstrates its accurate descriptive ability.

Brams' and Arrow's Work

Brams views coalition formation in a political context which is related specifically to voting. He suggests that coalitions form for the sole purpose of winning. A player searches not for a partner who has a similar preference ordering but who provides that player with a greater chance of winning. Indeed, Brams advocates Riker's [33] size concept that the winning coalitions will be no larger than minimum size. Furthermore, Brams disagrees with coalitions formed by minimum ideological distance because of circular reasoning:

> Ideological compatibility is determined from the history of previous party alignments, which is information then used to predict what coalitions *will* form that are ideologically compatible. [22]

The principle that coalitions form to win (and then only minimally) appears to be a valid and logical argument in the voting context described by Brams. Clearly, when one party or group merges with another party, it probably does so in order to obtain a majority of votes. However, the winning coalition concept loses its intrinsic value in the broader nonquantitative political context used in this paper. Specifically, a player simply does not just win or lose. A player may, as a result of forming a coalition, obtain an equilibrium more preferable in his preference vector, relative to the other players' preference vectors. To say this player "wins" would not be as accurate as saying that his situation improves. Thus, although it seems logical to assume coalitions form to win in voting situations as described by Brams, this reasoning does not appear to be applicable to the coalition algorithms given in this paper.

Brams also argues that coalitions do not form to minimize ideological differences because these coalitions are based on previous party alignments. At first glance, Brams appears to directly contradict the ordinal outcome group metric described in this paper which indicates that players with preference vectors arranged in a similar manner are more likely to merge than those with preference vectors arranged in a dissimilar manner. However, a closer inspection of the algorithms reveals that, although the players may order their preferences according to ideology, they more probably order their preferences specifically according to the problem at hand. For example, two ideologically dissimilar players, the British and PF, are seen to be more likely to coalesce than other players in the Zimbabwe conflict. The PF and the British order their vectors in a similar manner because their goal of converting Zimbabwe-Rhodesia to a black majority rule government is the same and not because both are of a particular political persuasion. Thus, because ideological similarity is not the sole reason for the construction of each player's preference vector, the algorithms described in this paper avoid this problem.

Arrow [23] has constructed the general possibility theorem which essentially states that the two axioms and five conditions he has postulated cannot all hold true at once for any coalition. Indeed, in their present state, the coalition algorithm given in this paper seems to violate Arrow's "condition of independence of irrelevant alternatives." This condition states that if an arrangement of outcome pairs for two different sets of preference vectors is exactly the same, then the two coalition preference vectors outcome pairs resulting from these two sets of preference vectors must be the same. For example, consider outcomes 3 and 2 in Cases 1 and 2 for players A, B, and C in Table XI. In Cases 1 and 2 the ordering of outcome 3 and outcome 2 for all players remains constant, which, according to Arrow's condition should result in two similar coalition orderings for 3 and 2. The coalition algorithm appears to contradict Arrow's condition because in Case 1 the coalition perference vector is nonordinal between outcomes 2 and 3, whereas in Case 2, outcome 3 is preferred to outcome 2. However, Case 1 and Case 2 are treated exactly the same in stability analysis and because of the UI given by 3 and the UC's 1, 2, and 3. The net effect of these UI's and UC's on a stability analysis of Case 1 would be to treat outcome 3 as preferred to outcome 2 and outcome 1 equally preferred to outcome 3. Thus, if outcome I were removed, Case I would simply dissolve into Case 2, and Arrow's condition is, in fact, avoided. Indeed, it presently appears as though all Arrow's five conditions are satisfied.

In order to satisfy Arrow's five conditions, however, the concept of nonordinality was introduced into the coalition algorithm. Specifically, Arrow's axiom constraining coalition analysis to transitive relations between outcomes was expanded to allow intransitivities into the coalition preference vector. For example, consider outcomes 288, 289, and 160 in Table IX. Outcome 288 is indifferent to outcome 160 as indicated by both their UC's and shown by their reverse ordering in the PF and British individual preference vectors. Similarly, outcome 160 is indifferent to outcome 289. If transitivity held, outcome 288 should have been indifferent to outcome 289. Inspection of the PF and British individual preference vectors shows, however, that outcome 288 is preferred to outcome 289 in the coalition preference vector. Therefore, intransitivity exists for these three outcomes and, in general, can be shown to exist for other outcomes in a coalition preference vector.

Allowing intransitivity into the coalition preference vector seems valid because it probably describes a player's preference ordering more realistically. For example, although an individual may prefer apples to oranges and oranges to grapes, he may or may not prefer apples to grapes. Assuming transitivity implies he must prefer apples to grapes, whereas the assumption of intransitivity allows for his preference ordering to be either way or indifferent between the two items. Thus intransitivity is probably a more fruitful assumption to make than transitivity.

In summary, discussion in this section has identified the work of other writers and how it relates to the present study. The basic difference between this paper and many other papers is that the former is ordinally based and the latter are cardinally based. Restrictions on coalitions proposed by other writers have also been shown to either not apply to the present study or to be satisfied by the algorithm.

TABLE XI
ARROW'S CONDITION OF THE INDEPENDENCE OF IRRELEVANT ALTERNATIVES

	Case 1				Case 2	
Player A	1	3	2		3	2
Player B	3	2	1		3	2
Player C	3	1	2		3	2
Coalition	1	2	3		3	2
		3		Ws		
	2	1		UC's		
	3		1	UC's		

VI. CONCLUSION

The coalition algorithm is presented in this paper to determine the overall ordinal preference vector for two or more players in a coalition. This algorithm is extremely practical and straightforward to use and appears to satisfy constraints posed by other authors on coalition formation. When the coalition algorithm is employed in conjunction with a suitable stability analysis algorithm such as the improved metagame analysis algorithm of Fraser and Hipel [1], the manner in which various coalitions can affect the possible equilibria to the conflict can be fully investigated. In other words, comprehensive sensitivity analyses can be executed for a given dispute due to the availability of the new coalition analysis algorithm.

Actual coalitions frequently arise in many real-world disputes, while in other situations considering possible coalitions is often necessary even though a coalition may not eventually form. Players involved in a current controversy can employ the procedures developed in this paper to ascertain which possible coalitions could benefit them the most and also which coalitions are most likely to form. By joining a suitable coalition, more preferable final resolutions may be possible for a given player. On the other hand, coalitions formed by other players could adversely affect the possible equilibria for a specified player. For the case of the Zimbabwe conflict, the coalition analysis suggested that of the three players in the conflict, the PF and British had more in common and therefore could possibly form a meaningful coalition. Furthermore, this application demonstrated that it is straightforward to consider coalitions formally in a given dispute, and consequently, the coalition algorithm should be useful for studying energy, environmental, military, business, and virtually any other type of conflict.

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