Graphing Calculator Programs For Instructional Data Diagnostics and Statistical Inference

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Abstract

A number of programs, written for the TI–83 Plus calculator, are demonstrated in this article to illustrate this graphing calculator's surprisingly advanced statistical capabilities: residual plots for analysis of variance, pairwise comparison in one–factor experimental design, statistical inference for simple linear regression and confidence intervals for contrasts used in experimental design. These and a number of other programs are available for download from my web page. Advancements in graphing calculator statistical programs, such as those described in this article, allow instructors and students in introductory applied graduate–level statistics courses to perform sophisticated statistical data diagnostic and inference procedures during class time in an ordinary class room and outside of a computer lab.

1 Introduction

The one thing that has always bothered me about using computer–based statistical software packages is the "disconnect" between the classroom lecture and the application of the computer software to the material being taught. Based on my experiences, unless instructor and student are both in a computer lab at the same time, any discussion involving the use of computer software is awkward at best. I believe hand–held calculators to be superior in a classroom setting to computer–based software in giving students a better immediate understanding of how to manipulate data.

I was able to teach an introductory *graduate* statistics course for biologists, based on the text by Rao (1998), relying almost exclusively on graphing calculators, without having to resort to the use of computer–based statistical software packages such as SAS, SPSS or Splus or Minitab or spreadsheets, such as Excel. The calculator programs given here are essentially a step above the calculator programs which support introductory undergraduate statistics courses, such as the TI–83 Graphing Calculator Manual (Neal, 2000) used to support The Basic Practice of Statistics (Moore 2000) or the Technology Guide used to support Brase and Brase's Understandable Statistics, where both guide and text are by Brase and Brase (1999a, 1999b).

For the first ten years of my sixteen–year teaching career, I used a variety of statistical software packages, like SAS and Minitab, in teaching introductory statistics courses. In fact, the initial use of the TI–83 was driven essentially by necessity–Purdue University North Central did not have easy access to SAS until recently. The TI-83 was to fill in the gap until SAS arrived, but because I found the calculator so advanced, it has more or less replaced SAS, even though SAS is now readily available at Purdue University North Central. I do now use SAS, but only in a secondary role to the TI–83 calculator, in the introductory graduate statistics course I teach.

This paper is intended for college faculty who teach introductory graduate courses in statistics, particularly for those who teach students in non-mathematical, non-statistical majors. Having said this, this paper may be of interest to college faculty and Advanced Placement high school teachers who teach introductory undergraduate courses in statistics. In time, the graduate level statistical topics covered in this paper may well start being taught at an introductory undergraduate level, in part because they are able to, due to the powerful instructional presentation features of the TI–83 calculator and other similar calculators.

The last ten years have seen a steady improvement in the complexity in the kinds of statistical problems that scientific/graphing calculators have been able to handle. Early versions of graphing calculators were really no more than glorified scientific calculators able to calculate various summary statistics such as the average, median, standard deviation and percentiles with some simple graphing capabilities. More recent versions are now able to not only perform various statistical inference procedures such as z–tests, 2–proportion z–tests and analysis of variance (ANOVA) procedures, but also can produce a variety of statistical plots such as scatter plots, histograms and box plots, such as explained in Morgan (1997).

Probably the most important change in graphing calculators has been their greatly expanded programming capabilities which has been used to advantage by myself and others such as Kelly (1998). Within the last couple of years, graphing calculators have introduced "flash–memory", to allow these calculators to be electronically improved with maintenance and feature upgrades, much like the operating system of a desk–top computer can be upgraded. Other improvements include various cables that allow programs to be easily transferred from one calculator to another and from microcomputers. This latter improvement now allows distance learning students to copy graphing calculator programs from the Internet, and so allow them to better participate in statistics courses that store such programs on the Internet. Graphing calculator programs stored on the Internet may also provide a viable alternative for those Internet students who do not have ready access to sophisticated statistical computer packages.

2 Graphing Calculator Programs

Four example TI–83 Plus programs, and their outputs, are demonstrated below.

- residual plots for analysis of variance
- pairwise comparison in one–factor experimental design
- statistical inference for simple linear regression
- confidence intervals for contrasts used in one–factor experimental design

These programs are written for a *graduate* level introductory course in statistics. Although they could be used to support topics taught at an undergraduate level, they actually are intended to support the same topics, more or less, but dealt with in greater depth. These and many other programs are alphabetically listed later on in this paper and are available for download at:

http://faculty.purduenc.edu/jkuhn/courses/other/TI-83/STAT503/503-TI-83.html

2.1 Residual Plots For Analysis of Variance

The program QQPLTANV creates a q–q plot to check for normality and the program EVPPLOT creates an $e \vee p$ plot to check for constant variance where both assumptions would be necessary to proceed with an analysis of variance. These programs would probably be used after a discussion on how to conduct an analysis of variance, to clarify when it would be possible to conduct such a statistical procedure.

In this example, the QQPLTANV and EVPPLOT programs are used on the following oxygen consumption rate of mice for various humidity levels data.

5%	7	7	15	11	9	10
10%	12	17	12	18	18	16
15%	14	18	18	19	19	17
20%	19	25	22	19	23	24
25%	7	10	11	15	11	14

Table 1. Oxygen Consumption Of Mice For Various Humidity Levels

Type the five sets of six data points, corresponding to each of the five humidity levels, into the first five lists of the calculator, L_1, ..., L_5.

- Press the program key, PRGM, choose the QQPLTANV program, followed by pressing ENTER, then press 5, where "5" is the number of treatments, followed by ENTER.
- In a similar way, enter PRGM EVPPLOT ENTER 5 ENTER.

The two programs produce the two residual plots given below: the q–q plot (on the left, which indicates normality) and $e \vee p$ plot (which indicates constant variance).

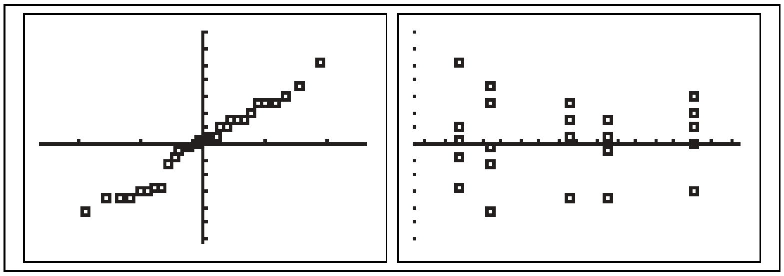


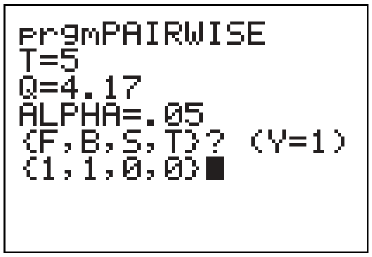
Figure 1. Residual Plots For Analysis of Variance

2.2 Pairwise Comparison In One–Factor Experimental Design

The program PAIRWISE performs four types of least significance difference (LSD) for pairwise comparisons in means tests: Fisher (F), Bonferroni (B), Scheffe (S) and Tukey (T). This program would probably be used if an analysis of variance test was found to be significant, to clarify how the various means differed from one another in a pairwise way.

In this example, the program PAIRWISE performs both Fisher's LSD test and Bonferroni's LSD test to decide if there are any significant pairwise differences in the mean oxygen rate of consumption for the i–th humidity and the mean oxygen rate of consumption for the j–th humidity, for $i, j = 1, \ldots, 5$, at a level of significance of $\alpha = 0.05$, for the oxygen consumption of mice for various humidity levels data given above.

The input for the Fisher and Bonferroni LSD pairwise tests, using the PAIRWISE program, appears on the calculator screen as given in the figure below.



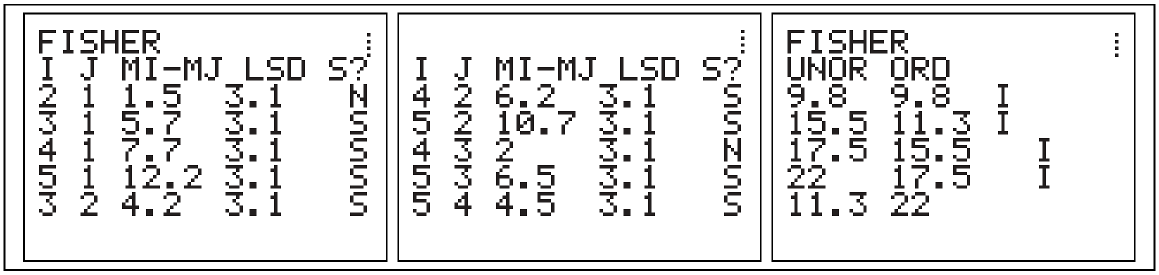
```
prgmPAIRWISE
T=5
Q=4.17
ALPHA=.05
{F,B,S,T}? (Y=1)
{1,1,0,0}█
```

Figure 2. Input For Fisher and Bonferroni LSD Pairwise Tests

There are five treatments, T=5, the value of the studentized range is 4.17, Q=4.17 (although, in this case, it is not necessary to know this value, since Tukey's test is not chosen), the level of significance is $\alpha = 0.05$ and {1,1,0,0} for {F,B,S,T}? means the first two (Fisher and Bonferroni) of the four possible least significant tests are chosen because "1" means "yes, do the test" and "0" means "do not do the test".

The results for the Fisher LSD pairwise tests are displayed in two different but complementary ways. These two different types of displays show all pairwise treatment means are significantly different from one another, except for the (5%, 25%) treatment pair and the (10%, 15%) treatment pair.

In the *third* section of the figure below, UNOR is the original unordered set of treatment means, (5%, 10%, 15%, 20%, 25%), and ORD is the ordered set of treatment means, (5%, 25%, 10%, 15%, 20%). *In particular, notice that the 25% mean, 11.3, is the second (2nd) smallest mean, after the 5% mean, 9.8.* In this last section, a column (or line or string) of "I"s indicate which treatments means are "tied together" or *not* significantly different from one another. For example, a column of "I"s ties the (9.8, 11.3) pair, or, equivalently, the (5%, 25%) pair, together and so this treatment pair mean difference is not significance.



```
FISHER
I J MI-MJ LSD S?
2 1 1.5  3.1  N
3 1 5.7  3.1  S
4 1 7.7  3.1  S
5 1 12.2 3.1  S
3 2 4.2  3.1  S
```

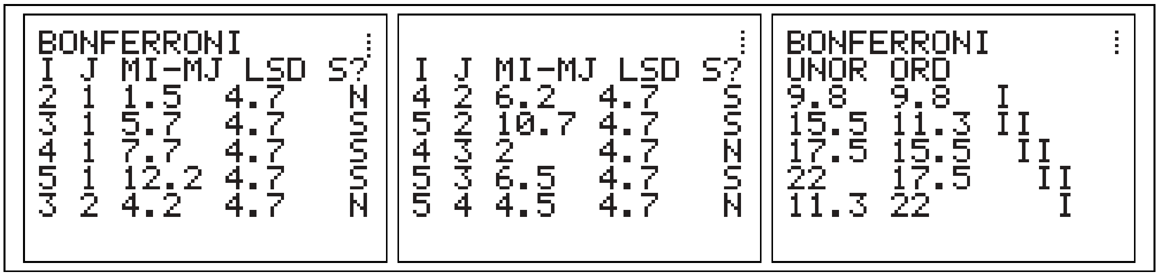
```
I J MI-MJ LSD S?
4 2 6.2  3.1  S
5 2 10.7 3.1  S
4 3 2    3.1  N
5 3 6.5  3.1  S
5 4 4.5  3.1  S
```

```
FISHER
UNOR ORD
9.8  9.8  I
15.5 11.3 I
17.5 15.5   I
22   17.5   I
11.3 22
```

Figure 3. Fisher LSD Pairwise Tests

In the first two sections of the figure above, I is the i–th *ordered* mean oxygen consumption observation, J is the j–th ordered observation, MI−MJ is the difference in the means, LSD is Fisher's least significant difference and S? asks whether the observed difference in means is “significant” or not significant (larger than the Fisher LSD or not).

The output for the Bonferroni LSD pairwise tests, given below, show four treatment pair mean differences are *not* significant: (9.8, 11.3), (11.3, 15.5), (15.5, 17.5) and (17.5, 22). Notice that there are more nonsignificant pairwise means in the conservative Bonferroni case, than in the previous Fisher case because the LSD cut–off level for significance in the Bonferroni case, 4.7, is larger (harder to achieve) than the LSD cut–off level for significance in the Fisher case, 3.1.



```
BONFERRONI     ⁝
I J MI-MJ LSD S?
2 1 1.5  4.7   N
3 1 5.7  4.7   S
4 1 7.7  4.7   S
5 1 12.2 4.7   S
3 2 4.2  4.7   N
```

```
               ⁝
I J MI-MJ LSD S?
4 2 6.2  4.7   S
5 2 10.7 4.7   S
4 3 2    4.7   N
5 3 6.5  4.7   S
5 4 4.5  4.7   N
```

```
BONFERRONI     ⁝
UNOR ORD
9.8  9.8  I
15.5 11.3 II
17.5 15.5  II
22   17.5   II
11.3 22      I
```

Figure 4. Bonferroni LSD Pairwise Tests

2.3 Statistical Inference In Simple Linear Regression

The program REGINF performs statistical inference for the slope, intercept and expected response in simple linear regression problems. This program would probably be used after a discussion on how to calculate a simple linear regression.

In this example, the program REGINF performs three statistical inference procedures for the expected reading ability response, $\mu(x)$, all for the reading ability versus level of illumination data given below. The program two–tailed tests if the expected reading ability is $\mu(x) = 75$ at a level of illumination of $x = 1.5$; it calculates a confidence interval for $\mu(x)$; and it calculates a prediction interval with $m = 1$ future value.

illumination, x	1	2	3	4	5	6	7	8	9	10
ability to read, y	70	70	75	88	91	94	100	92	90	85

Table 2. Reading Ability Versus Level Of Illumination

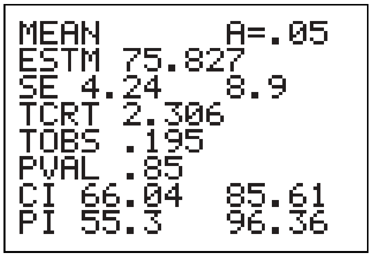
Type illumination levels and reading ability levels into L_1 and L_2, respectively.

- Enter PRGM REGINF ENTER.
- Type {0,0,1,1} for {B0,B1,MN,PI}?, which says do *not* perform statistical inference for either the intercept, β_0 (B0), or slope, β_1 (B1), but *do* perform

statistical inference (test and determine a confidence interval) for the mean $\mu(x)$ (MN) and *do* calculate a prediction interval (PI) for the mean $\mu(x)$.

- Type 75, the null expected reading ability response hypothesis, in response to "$\mu(x) =$".
- Press 1, to calculate the prediction interval for one (as opposed to two or more) future sample value(s), in response to "M=".
- Type 1.5, to test the expected reading ability response at an illumination level of 1.5, in response to "X=".
- Type the level of significance, 0.05, for "$\alpha =$".

The output is given below. The test fails to reject the null mean expected reading ability response at $\mu(x) = 75$ because of a large p–value of 0.85. The confidence interval and prediction intervals are (66.04, 85.61) and (55.3, 96.36), respectively.



```
MEAN      A=.05
ESTM 75.827
SE 4.24   8.9
TCRT 2.306
TOBS .195
PVAL .85
CI 66.04  85.61
PI 55.3   96.36
```

Figure 5. Inference For $\mu(x)$

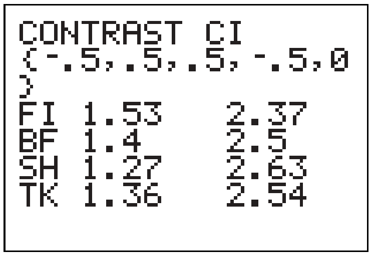
The ESTM is $\hat{\mu}(75)$, the estimated expected mean reading ability using a simple linear regression at $x = 1.5$, SE is the standard error (in both a confidence interval and a prediction interval situation, respectively), TCRT is the critical value of the t–distribution at 2.5% (half of 5%, since it is a two–tailed test), TOBS is the observed value of the test statistic, PVAL is the p–value, CI is the confidence interval and PI is the prediction interval.

2.4 Confidence Intervals For Contrasts Used In One–Factor Experimental Design

Questions from statistics texts often require the student to work, not from a data set, but from a *summary* of data. The program CSTCISTA determines confidence intervals for contrasts used in one–factor experimental design, when the data has been summarized (another program, CSTCI, determines the confidence intervals from the data set). This program would probably be used after a discussion on analysis of variance, to clarify in what (not necessarily pairwise) way the various means differed from one another.

Given $t = 5$ treatments, where the means are $\{2.5, 3.1, 4.5, 1.2, 6.5\}$, the sample sizes are $\{5, 5, 5, 5, 5\}$ and the standard deviation is $s = 0.45$, this program calculates four confidence intervals (Fisher, Bonferroni, Scheffe and Tukey) for the contrast $\theta = \frac{1}{2}(-\mu_1 + \mu_2 + \mu_3 - \mu_4) + (0)\mu_5$, assumed to be one of $k = 4$ confidence intervals, where the studentized range statistic is $q = 4.17$ and where $\alpha = 0.05$.

The program returns the following four 95% confidence intervals, where the FI is Fisher, BF is Bonferroni, SH is Scheffe and TK is Tukey.



```
CONTRAST CI
{-.5,.5,.5,-.5,0
}
FI 1.53   2.37
BF 1.4    2.5
SH 1.27   2.63
TK 1.36   2.54
```

Figure 6. Confidence Intervals For Contrasts

3 Alphabetical Listing of TI–83 Plus Programs

An alphabetical list of all of the TI–83 Plus programs I created along with a brief description of each program, is given below.

PRGM	Description
CCV	critical contrast value
CCVDISP (CCV)	critical contrast value display
CORR	Fisher correlation test/CI, data
CORRSTAT	Fisher correlation test/CI, stats
CORRZERO	exact correlation test/CI, data
CORRZSTA	exact correlation test/CI, stats
CSTCI	contrast CI, from data
CSTCISTA	contrast CI, from stats
CSTDISP (CSTCI, CSTCISTA)	contrast CI display
EVPPLOT	$e \vee p$ plot, ANOVA
EVXPLOT	$e \vee x$ plot, linear regression
EVYPLOT	$e \vee \hat{y}$ plot, linear regression
EVYPTQRG	$e \vee \hat{y}$ plot, quadratic regression
HYPCDF	hypergeometric cdf
HYPPDF	hypergeometric pdf
INVCHI2	χ^2 percentile
INVF	F percentile
INVT	t percentile
PAIRCMPR (PAIRWISE, PAIRSTAT)	pairwise contrast comparison
PAIRPICT (PAIRWISE, PAIRSTAT)	pairwise contrast picture (display)
PAIRSTAT	pairwise contrast by stats
PAIRWISE	pairwise contrast by data
PI	prediction interval by data
PIDISP (PI, PISTAT)	prediction interval display
PISTAT	prediction interval by stats
QQPLTANV	q–q plot for ANOVA
QQPLTQRG	q–q plot for quadratic regression
QQPLTREG	q–q plot for linear regression
REGANOVA	regression test β_1 by ANOVA
REGCALC (REGINF)	regression inference calculation, β_0, β_1, μ
REGDISP (REGINF)	regression inference display, β_0, β_1, μ
REGINF	regression inference, β_0, β_1, μ
REGSCIPI	simultaneous regression CI/PI
SCTPLOT	scatter plot

Table 3. List of Available Calculator Programs

Notice that "CCVDISP (CCV)", for example, means that the program CCVDISP is a *subroutine* of the program CCV. It is *not* possible to run the subroutine CCVDISP *directly*; subroutine CCVDISP is accessed indirectly when running program CCV.

4 Discussion

The main disadvantages of graphing calculators are they are small and can be slow. Small screens restrict the amount of information that can be displayed at one time. Small keyboards mean programming or accessing information can be slow and/or difficult. Rather than the microseconds it might take a typical desk–top computer, it might take the calculator seconds, or tens of seconds (but not minutes) to complete some of the programs demonstrated above. Flash memory and cables that allow calculators to link to microcomputers, however, seem to help alleviate some of these problems.

My experience indicates there are a number of advantages of using graphing calculators instead of computer–based statistical software.

- Students get hands–on experience manipulating data sets along with the instructor, inside an ordinary class room setting and outside a computer lab.
- Realistic data sets can be explored and analysed in real–time, during a test situation.
- Instructors and teaching assistants can help students with homework assignments with realistic data sets away from a computer monitor.
- The programming capabilities of the TI–83 plus are surprisingly advanced. It seems very possible to write programs for this calculator that are even more advanced than those given in this paper.

There might be some concern that students who have had previous training on the TI–83 calculator have an advantage over those who have not had such previous training. The "official" calculator used at Purdue University North Central is the TI–83 calculator. That is, although students can use any calculator they wish, only the TI–83 calculator is used and supported by all instructors and tutors in the mathematics, statistics and physics section (and, to a lesser extent, in all of the other sections on campus) at Purdue University North Central. This has been a section policy for a few years now. By the time students get to this introductory graduate biology course in statistics, they almost certainly will be familiar with this particular calculator. Even those who somehow start this course without having used the TI–83 in a previous course, tend to pick up on the calculator quickly because they can seek help from almost anyone else in the class or from the tutors who are all skilled in the use of the calculator. More than this, it seems to me that most students have had some experience with some type of Texas Instrument calculator from their high school days.

I have not done a statistical study, but if one was done, it would probably be interesting in knowing by how much a student's understanding (as measured by homework, quiz and final exam scores) of the concepts in the course material was influenced by using either the calculator or using a statistical software package.

There is not as much of a difference between the calculator and the statistical software as might at first be imagined. For instance, consider the residual $e \vee p$ plot example above. For the calculator, as explained, the data is first typed into the five lists of the calculator and then the key strokes PRGM (ENTER), 5 (for the number of treatments) and then ENTER to display the plot. For the computer, using, say, the SAS computer package, the following code would be used:

```
DATA OXY_COMP;
    INPUT HUM $ OXY;
DATALINES;
5 7
5 7
and rest of data
;
PROC GLM DATA=OXY_COMP;
    CLASS HUM;
    MODEL OXY = HUM;
    PLOT OUT=PLOT R=RESID;
RUN;
PROC PLOT DATA=PLOT;
    PLOT RESID*HUM / VREF=0;
RUN;
```

In this particular case, the calculator does what the computer software does and is easier to use than the computer software.

The calculator does have a number of preset programs for statistical (and related probability) analysis, grouped mostly in three key locations: STAT PLOT, STAT and 2nd DISTR. The STAT PLOT key gives six plots, including a scatter plot, line plot, histogram, a couple of box plots and a q–q plot for simple linear regression. The STAT key is the most powerful key. It has three menus: EDIT for entering, sorting and clearing data; a CALC key which gives a variety of one and two summary statistics, as well as various regressions (linear, quadratic, cubic, and so on); and a TESTS key which performs a variety of simple one and two sample tests, as well as a simple linear regression test and an ANOVA test. I do not believe any of the programs I have written duplicate any program functions already provided on the calculator.

My programs either compliment what is already on the calculator or provide more advanced statistical tests or displays.

Software for downloading and installing TI–83 programs off of the internet, as well as the special computer–to–calculator cable, can be obtained from the following web location.

http://education.ti.com/

Once instructors install a copy of the programs onto their calculators, they can give their students the option of doing the same or they can simply pass these programs from their calculator to the students' calculator via the cable link that is provided with every new calculator.

5 Summary

A number of programs, written for the TI–83 Plus calculator, have been demonstrated in this article to illustrate this graphing calculator's advanced statistical capabilities. I hope this article encourages other educators to not only recognize the value of using graphing calculators in applied graduate level statistics courses but also to create programs to continue to improve the statistical capabilities of these calculators.

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References

Brase, D.H. and Brase, C.P., (1999a), *Understandable Statistics*, Boston, MA: Houghton Mifflin Company.

Brase, D.H. and Brase, C.P., (1999b), *Technology Guide for Understandable Statistics*, Boston, MA: Houghton Mifflin Company.

Kelly, B., (1998), *Exploring Statistics With The TI–83*, Burlington, Ontario, Canada: Brendan Kelly Publishing Company.

Moore, D.S., (2000), *The Basic Practice of Statistics*, New York, NY: W.H. Freedman.

Morgan, L., (1997), *Explorations: Statistics Handbook For The TI–83*, Texas Instruments Incorporated.

Neal, D., (2000), *TI–83 Manual for The Basic Practice of Statistics*, New York, NY: W.H. Freedman.

Rao, P.V., (1998), *Statistical Research Methods in the Life Sciences*, Pacific Grove, CA: Brooks/Cole Publishing Company.

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