Chapter 9

Estimating the Value of a Parameter Using Confidence Intervals

We look at both point and *confidence interval (CI)* estimates of three parameters: mean, μ, proportion, p, and standard deviation, σ. A CI estimate provides a range of values, or an interval of values, that, together, act as an estimate for a parameter.

9.1 The Logic in Constructing Confidence Intervals for a Population Mean When the Population Standard Deviation is Known

The $(1-\alpha)\cdot 100\%$ confidence interval for μ with known σ is called a *z-interval*[1]:

$$\bar{x} \pm z_{\frac{\alpha}{2}} \left(\frac{\sigma}{\sqrt{n}} \right).$$

and used when either underlying distribution is normal with no outliers[2] or if simple random sample size large ($n \geq 30$).

Exercise 9.1 (The Logic in Constructing Confidence Intervals for a Population Mean When the Population Standard Deviation is Known)

[1]It very rarely occurs population σ is known when population mean μ is unknown. In other words, this formula is very rarely used in practice.

[2]Normality and outliers are verified using a normal probability plot and boxplot, respectively. If nonnormal or outliers exist, try to increase sample size to $n > 30$, or, if not possible, use *nonparametric* statistical methods that do not require normality and are resistant to outliers.

1. *Estimates for population average bluejay weight.*
   Average weight of a simple random sample of 40 bluejays is $\bar{x} = 5.24$ grams with a (known) standard deviation of $\sigma = 0.25$ grams.

   (a) *Point estimate.*
   Point estimate of *population* (actual, true) weight of *all* bluejays, μ, is $\bar{x} =$ (choose one) **0.25 / 5.24 / 40**.
   Statistic $\bar{x} = 5.24$ probably does not exactly equal unknown parameter μ.

   (b) *Confidence interval (CI) using TI-84+.*
   A 95% CI for average weight of all bluejays, μ, is (choose closest one) $\mathbf{(5.1625, 5.3175)}$ / $\mathbf{(5.175, 5.305)}$ / $\mathbf{(5.1625, 5.5175)}$,
   where this *interval* includes not only smallest possible average weight of 5.1625 grams and largest possible average weight of 5.3175 grams, but also other average weights in between these two extremes such as point estimate, $\bar{x} = 5.24$. Length of this CI is $L = 5.3175 - 5.1625 = 0.1550$.
   (STAT TESTS ZInterval... Stats 0.25 5.24 40 0.95 Calculate)

   (c) *CI using TI-84+.*
   A 90% CI for average weight of all bluejays, μ, is (choose closest one) $\mathbf{(5.1625, 5.3175)}$ / $\mathbf{(5.175, 5.305)}$ / $\mathbf{(5.1831, 5.2969)}$,
   where, notice, length of this CI is $L = 5.305 - 5.175 = 0.1300$.
   (STAT TESTS ZInterval... Stats 0.25 5.24 40 0.90 Calculate)

   (d) *CI using TI-84+.*
   A 85% CI for average weight of all bluejays, μ, is (choose closest one) $\mathbf{(5.1625, 5.3175)}$ / $\mathbf{(5.175, 5.305)}$ / $\mathbf{(5.1831, 5.2969)}$,
   where, notice, the length of this CI is $L = 5.2969 - 5.1831 = 0.1138$.
   (STAT TESTS ZInterval... Stats 0.25 5.24 40 0.85 Calculate)

   (e) *Comparing CI lengths.*
   Length of 95% CI for μ, $L = 0.1550$, is (choose one)
   longer than / same length as / shorter than
   length of 90% CI for μ, $L = 0.1300$, which is (choose one)
   longer than / same length as / shorter than
   length of 85% CI for μ, $L = 0.1138$.
   Increasing confidence increases CI length.

   (f) *Margin of error.*
   Half of length, L, is margin of error, $E = \frac{L}{2}$.
   Consequently, for 95% CI,
   $E = \frac{L}{2} = \frac{5.3175-5.1625}{2} =$ (circle one) **0.0775 / 0.0650 / 0.0569**,
   and for 90% CI,
   $E = \frac{L}{2} = \frac{0.1300}{2} =$ (circle one) **0.0775 / 0.0650 / 0.0569**,
   and for 85% CI,
   $E = \frac{L}{2} = \frac{0.1138}{2} =$ (circle one) **0.0775 / 0.0650 / 0.0569**.

(g) *Other ways of writing confidence intervals.*
Different possible ways of writing 95% CI include (choose *one or more*!)

i. $\mathbf{(5.1625, 5.3175)}$
ii. $\mathbf{(5.24 - 0.0775, 5.24 + 0.0775)}$
iii. $\mathbf{5.24 \pm 0.0775}$

where $\bar{x} = 5.24$ is *point estimate* and $E = 0.0775$ is margin of error.
In a similar way,
90% CI is (choose one)
$\mathbf{5.24 \pm 0.0775}$ / $\mathbf{5.24 \pm 0.0650}$ / $\mathbf{5.24 \pm 0.0569}$
and 85% CI is (choose one)
$\mathbf{5.24 \pm 0.0775}$ / $\mathbf{5.24 \pm 0.0650}$ / $\mathbf{5.24 \pm 0.0569}$

(h) *CI using formula, a first look.*
Since $\bar{x} = 5.24$, $\sigma = 0.25$ and $n = 40$, the 95% CI for μ is

$$\bar{x} \pm z_{\frac{\alpha}{2}} \left(\frac{\sigma}{\sqrt{n}} \right) =$$

the *incomplete* answer (choose one)

i. $\mathbf{5.24 \pm z_{\frac{\alpha}{2}} \times \frac{0.25}{\sqrt{32}}}$,
ii. $\mathbf{40 \pm z_{\frac{\alpha}{2}} \times \frac{5.24}{\sqrt{0.25}}}$,
iii. $\mathbf{5.24 \pm z_{\frac{\alpha}{2}} \times \frac{0.25}{\sqrt{40}}}$,

where, once *critical value* $z_{\frac{\alpha}{2}}$ is known, is $(5.1625, 5.3175) = 5.24 \pm 0.0775$.
In a similar way,
90% CI is (choose one)
$\mathbf{5.24 \pm z_{\frac{\alpha}{2}} \times \frac{0.25}{\sqrt{40}}}$ / $\mathbf{5.24 \pm z_{\frac{\alpha}{2}} \times \frac{5.24}{\sqrt{40}}}$ / $\mathbf{5.24 \pm z_{\frac{\alpha}{2}} \times \frac{0.25}{\sqrt{50}}}$
and 85% CI is (choose one)
$\mathbf{5.24 \pm z_{\frac{\alpha}{2}} \times \frac{0.25}{\sqrt{40}}}$ / $\mathbf{5.24 \pm z_{\frac{\alpha}{2}} \times \frac{5.24}{\sqrt{40}}}$ / $\mathbf{5.24 \pm z_{\frac{\alpha}{2}} \times \frac{0.25}{\sqrt{50}}}$
All three CIs same except three critical values, $z_{\frac{\alpha}{2}}$, are different.

(i) *CI using formula: calculating critical value, $z_{\frac{\alpha}{2}}$.*

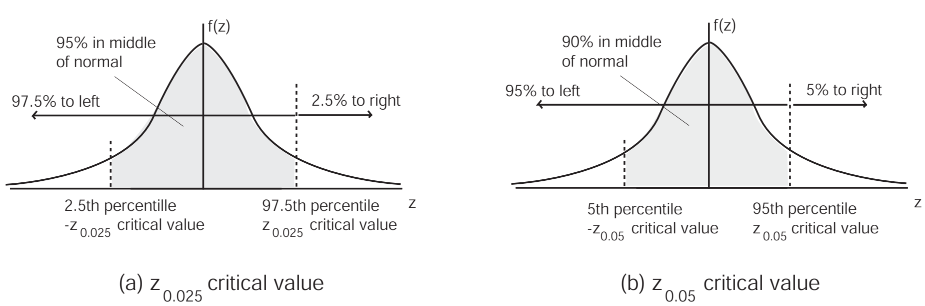

Figure 9.1 (Critical values.)

Critical value for 95% $= (1-\alpha)\cdot 100\% = (1-0.05)\cdot 100\%$ CI is
$z_{\frac{\alpha}{2}} = z_{\frac{0.05}{2}} = z_{0.025} =$ (circle one) **1.96 / 1.645 / 1.44**.
(Calculate 97.5th percentile[3]: 2nd DISTR 3:invNorm(0.975) ENTER.)
Critical value for 90% $= (1-\alpha)\cdot 100\% = (1-0.10)\cdot 100\%$ CI is
$z_{\frac{\alpha}{2}} = z_{\frac{0.10}{2}} = z_{0.05} =$ (circle one) **1.96 / 1.645 / 1.44**.
(2nd DISTR 3:invNorm(0.95) ENTER.)
Critical value for 85% $= (1-\alpha)\cdot 100\% = (1-0.15)\cdot 100\%$ CI is
$z_{\frac{\alpha}{2}} = z_{\frac{0.15}{2}} = z_{0.075} =$ (circle one) **1.96 / 1.645 / 1.44**.
(2nd DISTR 3:invNorm(0.925) ENTER.)

(j) *CI using formula.*
A 95% CI for average weight of all bluejays, μ, is $\bar{x} + z_{\frac{\alpha}{2}}\left(\frac{\sigma}{\sqrt{n}}\right) =$

i. $\mathbf{5.24 \pm 1.96 \times \frac{0.25}{\sqrt{40}} \approx (5.1625, 5.3175)}$
ii. $\mathbf{5.24 \pm 1.645 \times \frac{0.25}{\sqrt{40}} \approx (5.175, 5.305)}$
iii. $\mathbf{5.24 \pm 1.44 \times \frac{0.25}{\sqrt{40}} \approx (5.1831, 5.2969)}$

and a 90% CI is (choose one)

i. $\mathbf{5.24 \pm 1.96 \times \frac{0.25}{\sqrt{40}} \approx (5.1625, 5.3175)}$
ii. $\mathbf{5.24 \pm 1.645 \times \frac{0.25}{\sqrt{40}} \approx (5.175, 5.305)}$
iii. $\mathbf{5.24 \pm 1.44 \times \frac{0.25}{\sqrt{40}} \approx (5.1831, 5.2969)}$

and an 85% CI is (choose one)

i. $\mathbf{5.24 \pm 1.96 \times \frac{0.25}{\sqrt{40}} \approx (5.1625, 5.3175)}$
ii. $\mathbf{5.24 \pm 1.645 \times \frac{0.25}{\sqrt{40}} \approx (5.175, 5.305)}$
iii. $\mathbf{5.24 \pm 1.44 \times \frac{0.25}{\sqrt{40}} \approx (5.1831, 5.2969)}$

(k) *Checking assumptions.*
True / **False**. Since a large enough simple random sample collected, $n = 40 > 30$, central limit theorem (CLT) allows use of *standard normal* in CI formula; in particular, for critical value, $z_{\frac{\alpha}{2}}$.

2. *Estimates for population average touch-sensitivity of blind.*
In a simple random sample of 32 blind people, average touch-sensitivity is $\bar{x} = 0.013$ with standard deviation $\sigma = 0.003$.

(a) *Point estimate*
Point estimate of *population* average touch-sensitivity, μ, is
$\bar{x} =$ (choose one) **0.003 / 0.013 / 32**.

[3]The $z_{0.025}$ critical value is equal to 97.5th percentile: 2.5% to *right* of $z_{0.025}$ critical value equals 97.5% to left of 97.5th percentile. Also, 97.5th percentile, rather than 95th percentile is calculated here. This is because 95% CI occupies *middle* portion of normal curve and so upper bound of CI must have 95% + 2.5% = 97.5 % to left. Refer to previous chapter on normal distribution.

(b) *95% CI*

i. *Using TI-84+.* The 95% CI for μ is (circle one)
(0.01196, 0.01404) / **(0.01296, 0.01304)** / **(0.01396, 0.01204)**.
(STAT TESTS ZInterval... Stats 0.003 0.013 32 0.95 Calculate)
So, 95% *confident* population parameter μ in (0.01196, 0.01404).

ii. *Using formula: critical value.*
Critical value for 95% = $(1-\alpha)\cdot 100\% = (1-0.05)\cdot 100\%$ CI for μ is
$z_{\frac{\alpha}{2}} = z_{\frac{0.05}{2}} = z_{0.025} =$ (circle one) **1.28** / **1.96** / **2.58**.
(Calculate 97.5th percentile: 2nd DISTR 3:invNorm(0.975) ENTER.)

iii. *Using formula.*
Since $\bar{x} = 0.013$, $\sigma = 0.003$ and $n = 32$, the 95% CI for μ is
$\bar{x} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = \bar{x} \pm z_{0.025} \frac{\sigma}{\sqrt{n}} =$ (circle one)
$\mathbf{0.013 \pm 1.28 \times \frac{0.003}{\sqrt{32}}}$ / $\mathbf{0.013 \pm 1.96 \times \frac{0.003}{\sqrt{32}}}$ / $\mathbf{0.013 \pm 2.58 \times \frac{0.003}{\sqrt{32}}}$
$\approx (0.01196, 0.01404)$

iv. *Length*, L, of 95% CI is
$L = 0.01404 - 0.01196 =$ (circle one) **0.00176** / **0.00208** / **0.00354**.
Half of length, margin of error,
$E = \frac{L}{2} =$ (circle one) **0.00104** / **0.00208** / **0.00354**.
Notice, margin of error also equals

$$E = z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 1.96 \times \left(\frac{0.003}{\sqrt{32}}\right) \approx 0.00104.$$

(c) *92% CI*

i. *Using TI-84+.* The 92% CI is (circle one)
(0.01207, 0.01393) / **(0.01377, 0.01401)** / **(0.01077, 0.01189)**.
(STAT TESTS ZInterval... Stats 0.003 0.013 32 0.92 Calculate)
So, 92% *confident* population parameter μ in (0.01207, 0.01393).

ii. *Using formula: critical value.*
Critical value for 92% = $(1-\alpha)\cdot 100\% = (1-0.08)\cdot 100\%$ CI for μ is
$z_{\frac{\alpha}{2}} = z_{\frac{0.08}{2}} = z_{0.04} =$ (circle one) **1.56** / **1.75** / **1.96**.
(Calculate 96th percentile: 2nd DISTR 3:invNorm(0.96) ENTER.)

iii. *Using formula.*
The 92% CI is
$\bar{x} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = \bar{x} \pm z_{0.04} \frac{\sigma}{\sqrt{n}} =$ (circle one)
$\mathbf{0.013 \pm 1.56 \times \frac{0.003}{\sqrt{32}}}$ / $\mathbf{0.013 \pm 1.75 \times \frac{0.003}{\sqrt{32}}}$ / $\mathbf{0.013 \pm 1.96 \times \frac{0.003}{\sqrt{32}}}$
$\approx (0.01207, 0.01393)$

iv. *Margin of error.*
$E = z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \approx 1.75 \times \left(\frac{0.003}{\sqrt{32}}\right) \approx$ **0.00093** / **0.00075** / **0.00098**.

(d) *Some comments*

i. *Checking assumptions.*
True / **False**. Since a large enough simple random sample was collected, $n = 32 > 30$, CLT allowed the use of standard normal in CI formula; in particular, for critical value, $z_{\frac{\alpha}{2}}$.

ii. **True** / **False**.
Manipulation of margin of error formula gives formula for sample size.

$$\begin{aligned} E &= z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \\ E\sqrt{n} &= z_{\frac{\alpha}{2}} \cdot \sigma \\ \sqrt{n} &= \frac{z_{\frac{\alpha}{2}} \cdot \sigma}{E} \\ n &= \left(\frac{z_{\frac{\alpha}{2}}\sigma}{E}\right)^2 \end{aligned}$$

3. *Estimates for population average Ph levels in soil, using raw data.*
Ph levels measured for $n = 28 < 30$ soil samples, chosen at random; $\sigma = 3.01$.

4.3	5	5.9	6.5	7.6	7.7	7.7	8.2	8.3	9.5
10.4	10.4	10.5	10.8	11.5	12	12	12.3	12.6	12.6
13	13.1	13.2	13.5	13.6	14.1	14.1	15.1		

(a) *Check assumptions (since $n = 28 < 30$): normality and outliers.*

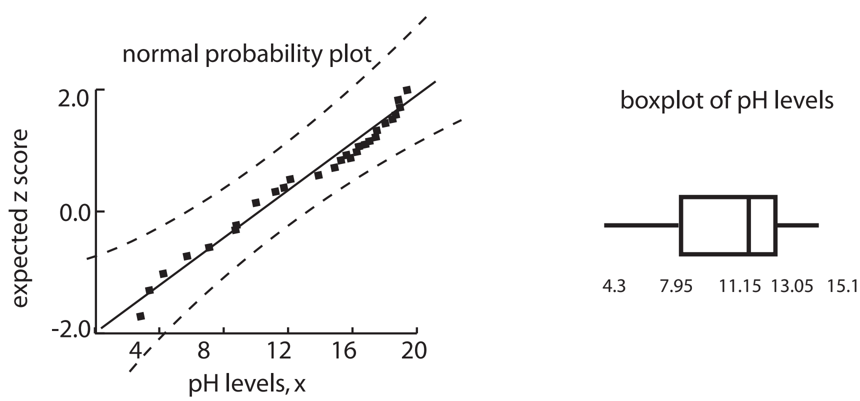

Figure 9.2 (Normal probability plot, boxplot for pH levels.)

i. *Data normal?*
Normal probability plot indicates pH level data **normal** / **not normal** because data within dotted bounds.

First clear $Y =$ and turn off all plots in 2nd $Y =$. Type data into L_1. 2nd STAT PLOT, ENTER to turn on first plot. Choose last graph on second line, then ZOOM ZoomStat ENTER for plot.

ii. *Outliers?*
Boxplot indicates **outliers** / **no outliers**.

As before, only choose first graph on second line, then ZOOM ZoomStat ENTER for plot.

(b) *Point estimate.*
Point estimate of *population* (actual, true) weight of *all* pH levels, μ, is $\bar{x} =$ (choose one) **10.55 / 11.55 / 12.55**.
STAT CALC ENTER ENTER (a 2nd time!); read $\bar{x}$.

(c) *68% CI*

i. *Using TI-84+.* The 68% CI for μ is (circle one)
$\mathbf{(9.33, 11.37)}$ / $\mathbf{(9.44, 11.67)}$ / $\mathbf{(9.99, 11.12)}$.
(Type data into L_1; then STAT TESTS ZInterval... Data 3.01 L_1 1 0.68 Calculate)
So, 68% *confident* population parameter μ in (9.99, 11.12).

ii. *Using formula: critical value.*
Critical value for 68% = $(1-\alpha)\cdot 100\% = (1-0.32)\cdot 100\%$ CI for μ is
$z_{\frac{\alpha}{2}} = z_{\frac{0.32}{2}} = z_{0.16} =$ (choose closest one) **1 / 2 / 3**.
(Calculate 84th percentile: 2nd DISTR 3:invNorm(0.84) ENTER.)

iii. *Using formula.*
The 68% CI for μ is
$\bar{x} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} =$ (circle one)
$\mathbf{10.55 \pm 1 \times \frac{3.01}{\sqrt{28}}}$ / $\mathbf{10.55 \pm 2 \times \frac{3.01}{\sqrt{28}}}$ / $\mathbf{10.55 \pm 3 \times \frac{3.01}{\sqrt{28}}}$
$\approx (9.99, 11.12)$

iv. *Margin of error.*
$E = z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \approx 1 \times \left(\frac{3.01}{\sqrt{28}}\right) \approx$ **0.45 / 0.57 / 0.64**.

(d) *95% CI*

i. *Using TI-84+.* The 95% CI for μ is (circle one)
$\mathbf{(9.33, 11.37)}$ / $\mathbf{(9.44, 11.67)}$ / $\mathbf{(9.99, 11.12)}$.
(Type data into L_1; then STAT TESTS ZInterval... Data 3.01 L_1 1 0.95 Calculate)
So, 95% *confident* population parameter μ in (9.44, 11.67).

ii. *Using formula: critical value.*
Critical value for 95% = $(1-\alpha)\cdot 100\% = (1-0.05)\cdot 100\%$ CI for μ is
$z_{\frac{\alpha}{2}} = z_{\frac{0.05}{2}} = z_{0.025} =$ (choose closest one) **1 / 2 / 3**.
(Calculate 97.5th percentile: 2nd DISTR 3:invNorm(0.975) ENTER.)

iii. *Using formula.*
The 95% CI for μ is
$\bar{x} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} =$ (circle one)
$\mathbf{10.55 \pm 1 \times \frac{3.01}{\sqrt{28}}}$ / $\mathbf{10.55 \pm 2 \times \frac{3.01}{\sqrt{28}}}$ / $\mathbf{10.55 \pm 3 \times \frac{3.01}{\sqrt{28}}}$
$\approx (9.44, 11.67)$

iv. *Margin of error.*
$E = z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \approx 2 \times \left(\frac{3.01}{\sqrt{28}}\right) \approx$ **1.14 / 2.23 / 6.69**.

(e) *Some comments*

i. Critical value, $z_{\frac{\alpha}{2}}$, is associated with empirical rule. Empirical rule states 68%, 95% and 99.7% of probability in normal distribution is within one (1), two (2) and three (3) SDs, respectively, of mean. For 68% CI, $z_{0.16} \approx 1$, and for 95% CI, $z_{0.025} \approx 2$. Consequently, for a 99.7% CI, (choose one) $\boldsymbol{z_{0.0015} = 1}$ / $\boldsymbol{z_{0.0015} = 2}$ / $\boldsymbol{z_{0.0015} = 3}$.

ii. Margin of error of 95% CI, $E = 1.14$, is more or less (circle one)
half as long as
same length as
twice as long as
as margin of error 68% CI, $E = 0.57$.
Also, a 99.7% CI is three times as long as a 68% CI.

4. *Sample size given margin of error and level of confidence: bluejay weights.* Sample size necessary to achieve a required margin of error, E, with a given level of confidence in a confidence interval determined using formula

$$n = \left(\frac{z_{\frac{\alpha}{2}}\sigma}{E}\right)^2.$$

(a) What sample size, n, required to estimate average bluejay weight, μ, to within margin of error $E = 0.08$ with 95% confidence? Assume $\sigma = 0.25$.

$$n = \left(\frac{z_{\frac{\alpha}{2}}\sigma}{E}\right)^2 = \left(\frac{z_{0.025}\sigma}{E}\right)^2 = \left(\frac{1.96 \cdot 0.25}{0.08}\right)^2 \approx 37.7 \approx$$

(circle one) **37** / **38** / **39**.

(Since 95% $= (1-\alpha) \cdot 100\% = (1-0.05) \cdot 100\%$, so use invNorm(0.975) for $z_{\frac{\alpha}{2}} = z_{\frac{0.05}{2}} = z_{0.025} \approx 1.96$.)

(b) *Increase margin of error, E.*
What sample size, n, required to estimate average bluejay weight, μ, to within margin of error $E = 0.16$ with 95% confidence? Assume $\sigma = 0.25$.

$$n = \left(\frac{z_{\frac{\alpha}{2}}\sigma}{E}\right)^2 = \left(\frac{z_{0.025}\sigma}{E}\right)^2 = \left(\frac{1.96 \cdot 0.25}{0.16}\right)^2 \approx 9.38 \approx$$

(circle one[4]) **9** / **10** / **11**.
When margin of error doubled, from $E = 0.08$ to $E = 0.16$, sample size (choose one) **quartered** / **halved** / **doubled** from $n = 38$ to $n = 10$.

(c) *Increase confidence,* $(1 - \alpha) \cdot 100\%$.
What sample size, n, required to estimate average bluejay weight, μ, to within margin of error $E = 0.16$ with 99% confidence? Assume $\sigma = 0.25$.

$$n = \left(\frac{z_{\frac{\alpha}{2}}\sigma}{E}\right)^2 = \left(\frac{z_{0.005}\sigma}{E}\right)^2 = \left(\frac{2.58 \cdot 0.25}{0.16}\right)^2 \approx 16.2 \approx$$

(circle one) **16** / **17** / **18**.
When confidence increased, from 95% to 99%, sample size (choose one) **decreases** / **remains same** / **increases** from $n = 10$ to $n = 17$.

(Since 99% $= (1-\alpha) \cdot 100\% = (1-0.01) \cdot 100\%$, so use invNorm(0.995) for $z_{\frac{\alpha}{2}} = z_{\frac{0.01}{2}} = z_{0.005} \approx 2.58$.)

[4] Always round *up*, never down, to satisfy margin of error and confidence conditions.

5. *Population, sample, statistic and parameter: CI for average touch-sensitivity.* Simple random sample of 32 totally blind people is taken. Assume $\sigma = 0.003$ and *population* $\mu = 0.013$ (*not sample* $\bar{x} = 0.013$!) touch-sensitivity.

   (a) *Population* $\mu = 0.013$ *touch-sensitivity*
   Population $\mu = 0.013$ is a **statistic** / **parameter**.
   Population μ **changes** / **remains same** for every *random* sample.
   Population μ **known** / **unknown** to us,
   but let's pretend for this question we do know it.

   (b) *Sample* $\bar{x}$ *touch-sensitivity*
   Sample $\bar{x}$ is a **statistic** / **parameter**.
   Sample $\bar{x}$ **changes** / **remains same** for every *random* sample.
   Sample $\bar{x}$ **known** / **unknown** to us:
   it may be $\bar{x} = 0.013$ for one sample, but $\bar{x} = 0.017$ for another sample.

   (c) A 95% CI for μ, if $\bar{x} = 0.013$, is
   $\bar{x} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 0.013 \pm 1.96 \frac{0.003}{\sqrt{32}} =$ (circle one)
   $\mathbf{(0.012, 0.014)}$ / $\mathbf{(0.013, 0.015)}$ / $\mathbf{(0.014, 0.016)}$.
   (STAT TESTS ZInterval... Stats 0.003 0.013 32 0.95 Calculate)
   This 95% CI (circle one) **contains** / **does not contain** $\mu = 0.013$.

   (d) A 95% CI for μ, if $\bar{x} = 0.017$, is
   $\bar{x} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 0.017 \pm 1.96 \frac{0.003}{\sqrt{32}} =$ (circle one)
   $\mathbf{(0.016, 0.018)}$ / $\mathbf{(0.017, 0.019)}$ / $\mathbf{(0.018, 0.020)}$.
   (STAT TESTS ZInterval... Stats 0.003 0.017 32 0.95 Calculate)
   This 95% CI (circle one) **contains** / **does not contain** $\mu = 0.013$.

   (e) If sample average touch-sensitivity, $\bar{x}$, changes, corresponding 95% CI, $\bar{x} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$, **changes** / **remains the same**. More than this, (circle one)

      i. *all* possible 95% CIs contain $\mu = 0.013$.
      ii. *none* of all possible 95% CIs contain $\mu = 0.013$.
      iii. ninety–nine percent of all possible 95% CIs contain $\mu = 0.013$, and so one percent of all possible 95% CIs do not contain $\mu = 0.013$.
      iv. ninety–five percent of all possible 95% CIs contain $\mu = 0.013$, and so five percent of all possible 95% CIs do not contain $\mu = 0.013$.

   This is demonstrated in figure below.

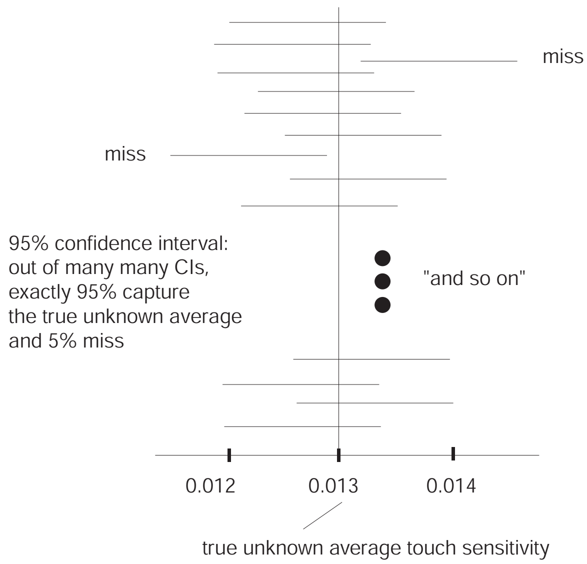

Figure 9.3 (Interpreting confidence intervals.)

(f) Choose true or false.

i. **True** / **False**. 95% *chance* (0.016, 0.018) contains μ.
ii. **True** / **False**. 95% *chance* (0.016, 0.018) contains $\bar{x} = 0.013$.
iii. **True** / **False**. 95% *confident* (0.016, 0.018) contains μ.
iv. **True** / **False**. 95% *confident* (0.016, 0.018) contains $\bar{x} = 0.013$.

9.2 Constructing Confidence Intervals for a Population Mean When the Population Standard Deviation is Unknown

The $(1-\alpha)\cdot 100\%$ confidence interval for μ with *un*known σ is called a *t-interval*:

$$\bar{x} \pm t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right).$$

and used when either underlying distribution is normal with no outliers or if simple random sample size large ($n \geq 30$).

Exercise 9.2 (Constructing Confidence Intervals for a Population Mean When the Population Standard Deviation is Unknown)

1. *Estimates for population average weight of PNC students.*
   Average weight of simple random sample of 11 PNC students is $\bar{x} = 167$ pounds with sample SD $s = 20.1$ pounds. Weights normally distributed, no outliers.

(a) *Point estimate.*
Point estimate of population weight of *all* students, μ, is
$\bar{x}$ = (choose one) **11** / **20.1** / **167**.
Also notice σ is *un*known and *estimated* by $s = 20.1$.

(b) *95% CI*

i. *Using TI-84+.* The 95% CI for μ is (circle one)
(143.5, 182.5) / **(151.5, 180.5)** / **(153.5, 180.5)**.
(STAT TESTS TInterval... Stats 167 20.1 11 0.95 Calculate)
So, 95% confident population parameter μ in (153.5, 180.5).

ii. *Using formula: degrees of freedom (df).*
df $= n - 1 = 11 - 1 =$ (circle one) **10** / **11**.

iii. *Using formula: critical value.*
Critical value 95% $= (1 - \alpha) \cdot 100\% = (1 - 0.05) \cdot 100\%$ CI, 10 df
$t_{\frac{\alpha}{2}} = t_{\frac{0.05}{2}} = t_{0.025} \approx$ (circle one) **1.28** / **2.23** / **2.58**.
(Calculate 97.5th percentile: 2nd DISTR invT(0.975, 10)
OR: PRGM INVT ENTER ENTER (again!) 10 ENTER 0.975 ENTER)

iv. *Using formula.*
The 95% CI for μ is
$\bar{x} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} =$ (circle one)
$\mathbf{20.1 \pm 167 \times \frac{2.23}{\sqrt{11}}}$ / $\mathbf{2.23 \pm 167 \times \frac{20.1}{\sqrt{11}}}$ / $\mathbf{167 \pm 2.23 \times \frac{20.1}{\sqrt{11}}}$
which equals (circle one)
$\mathbf{20.1 \pm 12.51}$ / $\mathbf{2.23 \pm 13.51}$ / $\mathbf{167 \pm 13.51} \approx (153.5, 180.5)$.

(c) *99% CI*

i. *Using TI-84+.* The 99% CI for μ is (circle one)
(147.8, 186.2) / **(151.5, 180.5)** / **(153.5, 180.5)**.
(STAT TESTS TInterval... Stats 167 20.1 11 0.99 Calculate)
So, 99% confident population parameter μ in (147.8, 186.2).

ii. *Using formula: degrees of freedom.*
df $= n - 1 = 11 - 1 =$ (circle one) **10** / **11**.

iii. *Using formula: critical value.*
Critical value 99% $= (1 - \alpha) \cdot 100\% = (1 - 0.01) \cdot 100\%$ CI, 10 df
$t_{\frac{\alpha}{2}} = t_{\frac{0.01}{2}} = t_{0.005} \approx$ (circle one) **1.28** / **2.23** / **3.17**.
(Calculate 99.5th percentile: PRGM INVT ENTER ENTER (again!) 10 ENTER 0.995 ENTER.)

iv. *Using formula.*
The 99% CI for μ is
$\bar{x} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} =$ (circle one)
$\mathbf{20.1 \pm 20.1 \times \frac{3.17}{\sqrt{11}}}$ / $\mathbf{3.17 \pm 167 \times \frac{20.1}{\sqrt{11}}}$ / $\mathbf{167 \pm 3.17 \times \frac{20.1}{\sqrt{11}}}$.
which equals (circle one)
$\mathbf{20.1 \pm 19.21}$ / $\mathbf{3.17 \pm 19.21}$ / $\mathbf{167 \pm 19.21} \approx (147.8, 186.2)$

v. *Margin of error (half of CI length)*

$$E = t_{\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}} \approx 3.17 \times \frac{20.1}{\sqrt{11}} \approx$$

(circle one) **11** / **12.5** / **19.2**.

vi. **True** / **False** There is a 99% *chance* population average weight, μ, falls in sample interval (147.8, 186.2).

2. *Confidence interval for average length of corn cobs, raw data.*
Corn cob lengths for $n = 15 < 30$ cobs, chosen at random, are noted.

$$18, 23, 24, 20, 21, 19, 27, 24, 19, 20, 25, 20, 18, 26, 20$$

(a) *Point estimate.*
Point estimate of population length of *all* cobs, μ, is
$\bar{x} =$ (choose one) **2.97** / **21.6** / **15**.
Also notice population SD in cob length, σ, is *un*known and *estimated* by
$s \approx$ (choose one) **2.97** / **21.6** / **15**..
(Type data into L_1, then STAT CALC 1-Var Stats L_1; read $\bar{x}$, s_X.)

(b) *Check assumptions (since* $n = 15 < 30$*): normality and outliers.*

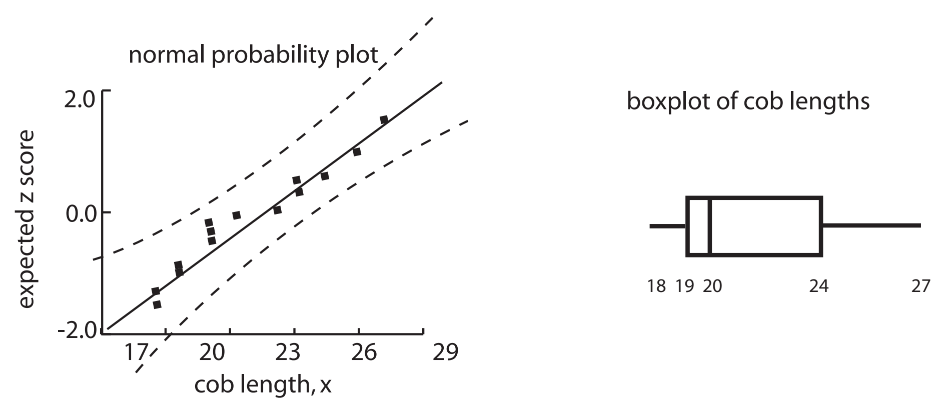

Figure 9.4 (Normal probability plot, boxplot for cob lengths.)

i. *Data normal?*
Normal probability plot indicates cob lengths **normal** / **not normal** because data within dotted bounds.
First clear $Y =$ and turn off all plots in 2nd $Y =$. Type data into L_1. 2nd STAT PLOT, ENTER to turn on first plot. Choose last graph on second line, then ZOOM ZoomStat ENTER for plot.

ii. *Outliers?*
Boxplot indicates **outliers** / **no outliers**.
As before, only choose first graph on second line, then ZOOM ZoomStat ENTER for plot.

(c) *95% CI*

i. *Using TI-84+.* The 95% CI for μ is (choose one)
(17.96, 21.24) / **(19.96, 22.24)** / **(19.96, 23.25)**.
(Type data into L_1; then STAT TESTS TInterval... Data L_1 1 0.95 Calculate)

ii. *Using formula: degrees of freedom (df).*
df $= n - 1 =$ (circle one) **15** / **14**.

iii. *Using formula: critical value.*
Critical value 95% $= (1-\alpha)\cdot 100\% = (1-0.05)\cdot 100\%$ CI, 14 df
$t_{\frac{\alpha}{2}} = t_{\frac{0.05}{2}} = t_{0.025} \approx$ (circle one) **1.76** / **2.15**.
(Calculate 97.5th percentile: PRGM INVT ENTER ENTER (again!) 14 ENTER 0.975 ENTER.)

iv. *Using formula.*
The 95% CI for μ is
$\bar{x} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} =$ (choose one)
$\mathbf{21.6 \pm 2.15 \times \frac{2.97}{\sqrt{15}}}$ / $\mathbf{21.6 \pm 2.15 \times \frac{3.97}{\sqrt{15}}}$ / $\mathbf{21.6 \pm 3.15 \times \frac{2.97}{\sqrt{15}}}$.

(d) *99% CI*

i. *Using TI-84+.* The 99% CI for μ is (choose one)
(19.23, 23.45) / **(19.96, 23.24)** / **(19.32, 23.88)**.
(Type data into L_1; then STAT TESTS TInterval... Data L_1 1 0.99 Calculate)

ii. *Using formula: degrees of freedom (df).*
The df, here, for 99% CI is (choose one) **same as** / **different from** degrees of freedom calculated for 95% CI above because same sample size is used in both cases.

iii. *Using formula: critical value.*
Critical value 99% $= (1-\alpha)\cdot 100\% = (1-0.01)\cdot 100\%$ CI, 14 df
$t_{\frac{\alpha}{2}} = t_{\frac{0.01}{2}} = t_{0.005} \approx$ (circle one) **1.76** / **2.98**.
(Calculate 99.5th percentile: PRGM INVT ENTER ENTER (again!) 14 ENTER 0.995 ENTER.)

iv. *Using formula.*
Thus, the 99% CI for μ is
$\bar{x} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} =$ (choose one)
$\mathbf{21.6 \pm 2.15 \times \frac{2.97}{\sqrt{15}}}$ / $\mathbf{21.6 \pm 2.15 \times \frac{3.97}{\sqrt{15}}}$ / $\mathbf{21.6 \pm 2.98 \times \frac{2.97}{\sqrt{15}}}$.
which equals (circle one)
$\mathbf{21.6 \pm 1.29}$ / $\mathbf{21.6 \pm 2.29}$ / $\mathbf{21.6 \pm 3.29} \approx (19.32, 23.88)$.

(e) *Some comments*

i. **True** / **False**. Long 99% CI better than shorter 95% CI in the sense we are more confident 99% contains or "captures" unknown parameter μ. However, 95% CI better than longer 99% CI in the sense, if unknown parameter μ is 95% interval estimate, we are more certain of location of this unknown parameter.

ii. Since sample size is small, we **can** / **cannot** use central limit theorem.

iii. Match columns.

terms	corn example
(a) population	**(a)** average length of 15 plants, $\bar{X}$
(b) sample	**(b)** average length of all plants, μ
(c) statistic	**(c)** lengths of all plants
(d) parameter	**(d)** observed lengths of 15 plants

terms	(a)	(b)	(c)	(d)
corn example				

3. *TI–84+: Confidence interval for mean μ, when σ unknown.*
In a study to determine effects of nitrates as meat preservatives, 15 data values, selected at random, are observed.

7251, 6871, 9632, 6866, 9094, 5849, 8957, 7978, 7468, 7064, 7494, 7883, 8178, 7523, 8724

For a 90% CI, type data into L_1; then STAT TESTS TInterval... Data L_1 1 0.99 Calculate. A pair of numbers, (7332.9, 8244.7), which are the upper and lower limits of the 90% CI, is returned.

4. *Percentages for t distribution: temperatures.*
Assume temperature, T, follows a t distribution with 4 degrees of freedom.

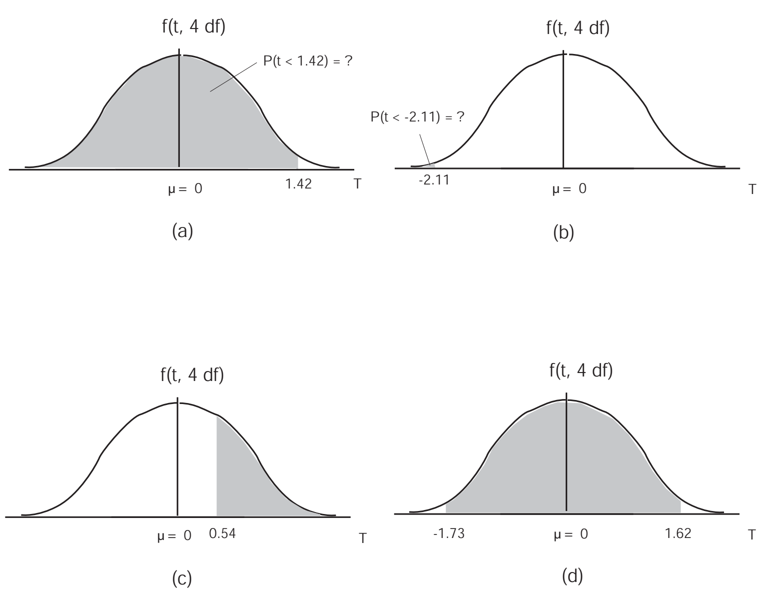

Figure 9.5 (Probabilities for t with 4 df.)

(a) Probability temperature is less than 1.42^o, is
$P(t < 1.42) =$ (circle one) **0.786** / **0.834** / **0.886** / **0.905**
(2nd DISTR tcdf(− 2nd EE 99 , 1.42 , 4) ENTER.)

(b) $P(t < -2.11) =$ (circle one) **0.023** / **0.051** / **0.124** / **0.243**.
(2nd DISTR tcdf(− 2nd EE 99 , −2.11 , 4) ENTER.)

(c) $P(t > 0.54) =$ (circle one) **0.309** / **0.356** / **0.435** / **0.470**.
(2nd DISTR tcdf(0.54 , 2nd EE 99 , 4) ENTER.)

(d) $P(-1.73 < t < 1.62) =$ (circle one) **0.647** / **0.734** / **0.801** / **0.830**.
(2nd DISTR tcdf(−1.73 , 1.62 , 4) ENTER.)

(e) t–distributions are (pick one) **skewed right** / **symmetric** / **skewed left**.

(f) Total area (probability) under any t–distribution curve is (circle one) **50%** / **75%** / **100%** / **150%**.

(g) Shape of t–distribution curve is (circle one) **triangular** / **bell–shaped** / **rectangular**.

(h) The t–distribution is centered at (circle one) $\boldsymbol{\mu = 0^o}$ / $\boldsymbol{\mu = 1.42^o}$.

(i) Since t–distribution is symmetric, (circle one) **25%** / **50%** / **75%** of temperatures are above (to right) of $\mu = 0^o$.

(j) **True** / **False** Probability temperature is *exactly* 1.42^o, say, is *zero*.
(2nd DISTR tcdf(1.42 , 1.42 , 4) ENTER.)

(k) *Comparing t to Z*

i. $P(t \leq 1.42^o) \approx 0.886$ **equals** / **does not equal** $P(Z < 1.42^o) \approx 0.922$ where t has 4 df and "Z" stands for the "standard normal".
(Is 2nd DISTR normalcdf(− 2nd EE 99 , 1.42 , 0, 1) ENTER ≈ 0.922
exactly equal to 2nd DISTR tcdf(− 2nd EE 99 , 1.42 , 4) ENTER.) ≈ 0.886 ?

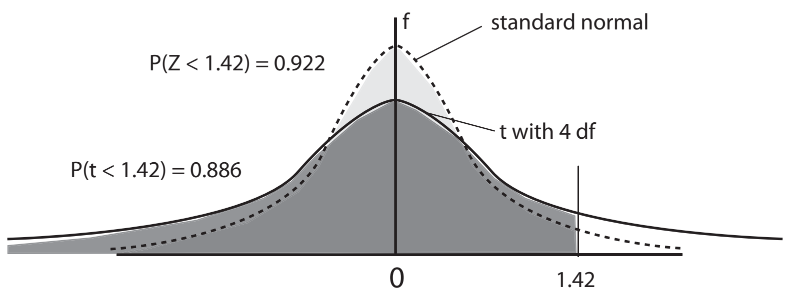

Figure 9.6 (Compare t to Z.)

ii. **True** / **False** $P(Z < 1.42^o) \approx P(t \leq 1.42^o)$ where t has 1000 df.
(Is 2nd DISTR normalcdf(− 2nd EE 99 , 1.42 , 0, 1) ENTER ≈ 0.922
approximately equal to 2nd DISTR tcdf(− 2nd EE 99 , 1.42 , 1000) ENTER.) ≈ 0.922

iii. **True** / **False**. The t distribution is a "flatter" version of standard normal. The larger the sample size, n, (or degrees of freedom, df) the less flat the t distribution becomes, the more like the standard normal it becomes.

5. *Percentiles for t distribution: temperatures.*
Assume temperature, T, follows a t distribution with 4 degrees of freedom.

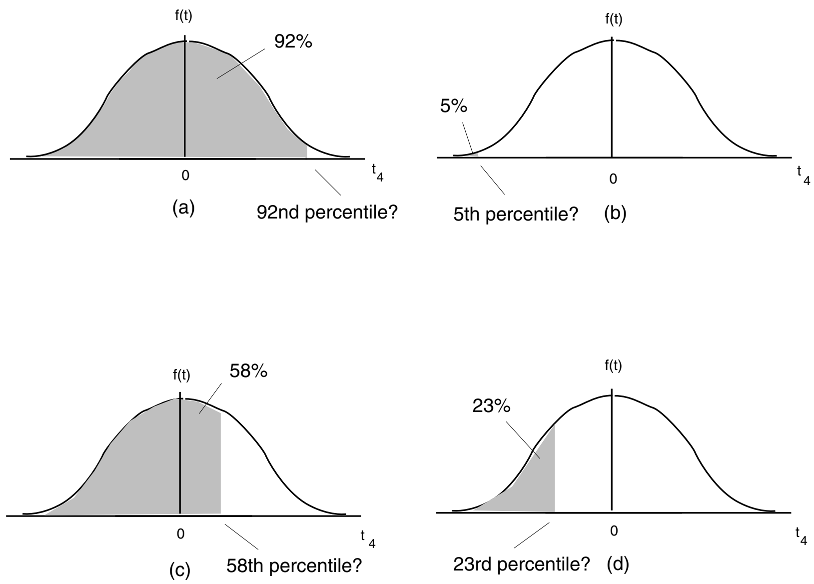

Figure 9.7 (Percentiles for t with 4 df.)

(a) The 92nd percentile is (circle one) $\mathbf{0.95^o}$ / $\mathbf{1.23^o}$ / $\mathbf{1.72^o}$ / $\mathbf{2.21^o}$.
(PRGM INVT ENTER ENTER 4 ENTER 0.92 ENTER.)

(b) The 5th percentile is (circle one) $\mathbf{-2.31^o}$ / $\mathbf{-2.13^o}$ / $\mathbf{-1.76^o}$ / $\mathbf{-0.76^o}$.
(PRGM INVT ENTER ENTER 4 ENTER 0.05 ENTER.)

(c) The 58th percentile is (circle one) $\mathbf{0.22^o}$ / $\mathbf{0.97^o}$ / $\mathbf{1.21^o}$ / $\mathbf{1.35^o}$.
(PRGM INVT ENTER ENTER 4 ENTER 0.58 ENTER.)

(d) The 23rd percentile is (circle one) $\mathbf{-1.58^o}$ / $\mathbf{-1.23^o}$ / $\mathbf{-0.82^o}$ / $\mathbf{-0.56^o}$.
(PRGM INVT ENTER ENTER 4 ENTER 0.23 ENTER.)

(e) *Percentiles and critical values*

i. The 95th percentile for t is equal to critical value $t_{0.05}$.
The 5th percentile for t is (circle *two*!) $\boldsymbol{t_{0.025}}$ / $\boldsymbol{-t_{0.05}}$ / $\boldsymbol{t_{0.95}}$.
The 99th percentile for t is (circle *two*!) $\boldsymbol{-t_{0.01}}$ / $\boldsymbol{t_{0.01}}$ / $\boldsymbol{-t_{0.99}}$.

ii. Probability (area) between $-t_{0.10}$ and $t_{0.10}$ is
(circle one) **0.80** / **0.09** / **0.88**.

6. *TI–84+: t Distribution.*

(a) Probability For t–distribution.
Probability t–distribution is less than 2.31 at 18 degrees of freedom, is
2nd DISTR tcdf(E99 , 2.31 , 18) ENTER. A probability of 0.9835 is returned.

(b) Percentile For t–distribution[5].
The 95th percentile of t–distribution with 18 degrees of freedom is
2nd DISTR invT(0.95,18)
OR: PRGM INVT ENTER ENTER 4 ENTER 0.95 ENTER A percentile of 1.734 is returned.

[5]If not done so, copy INVT program off of my web site and install it into your TI–84+ calculator.

9.3 Confidence Intervals for a Population Proportion

The confidence interval for proportion p from a binomial distribution is

$$\hat{p} \pm z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},$$

where we assume a large simple random sample has been chosen and $n\hat{p}(1-\hat{p}) > 10$. Sometimes, notation $\hat{q} = 1-\hat{p}$ is used. If sampled from finite population, $n \leq 0.05N$.

Exercise 9.3 (Confidence Intervals for a Population Proportion)

1. *Confidence interval (CI) for proportion, p, of purchase slips made with Visa.* It is found 54 of 180 (or $\hat{p} = \frac{54}{180} = 0.3$) randomly selected from 100,000 credit card purchase slips are made with Visa. Calculate a 95% CI of proportion p of purchase slips made with Visa.

   (a) *Check assumptions.*
   Since $n\hat{p}(1-\hat{p}) = 180(0.3)(0.7) = 37.8 > 10$, and $n = 180 < 0.05(100000) = 5000$, assumptions (choose one) **have** / **have not** been satisfied and so it is appropriate $\hat{p} \pm z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ estimate parameter p.

   (b) *Using TI-84+.* The 95% CI for p is (circle one)
   $\mathbf{(0.333, 0.367)}$ / $\mathbf{(0.273, 0.367)}$ / $\mathbf{(0.233, 0.367)}$.
   (STAT TESTS 1–PropZInt... 54 180 0.95 Calculate)
   So, 95% confident population parameter p in (0.233, 0.367).

   (c) *Using formula: critical value.*
   Critical value for $95\% = (1-\alpha)\cdot 100\% = (1-0.05)\cdot 100\%$ CI is
   $z_{\frac{\alpha}{2}} = z_{\frac{0.05}{2}} = z_{0.025} =$ (circle one) **1.96** / **1.645** / **1.44**.
   (2nd DISTR 3:invNorm(0.975) ENTER.)

   (d) *Using formula.*
   The 95% CI for p is
   $\hat{p} \pm z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} =$ (circle one)
   $\mathbf{0.7 \pm 0.3 \times \sqrt{\frac{0.3(0.7)}{180}}}$ / $\mathbf{0.5 \pm 0.3 \times \sqrt{\frac{0.3(0.7)}{180}}}$ / $\mathbf{\frac{54}{180} \pm 1.96 \times \sqrt{\frac{0.3(0.7)}{180}}}$
   $\approx (0.233, 0.367)$

2. *More questions*

   (a) *99% CI of proportion of purchase slips made with Visa.*
   It is found 54 of 180 (or $\hat{p} = \frac{54}{180} = 0.3$) randomly selected from 100,000 credit card purchase slips are made with Visa. The 99% CI of proportion p of purchase slips made with Visa is
   $\hat{p} \pm z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} =$ (circle *one or more*!)

i. $0.3 \pm 2.58 \times \sqrt{\frac{0.3(0.7)}{180}}$

ii. $0.3 \pm 2.58(0.034)$

iii. $(0.212, 0.388)$

(STAT TESTS 1–PropZInt... 54 180 0.99 Calculate;

For $z_{\frac{\alpha}{2}} = z_{\frac{0.01}{2}} = z_{0.005}$, use 2nd DISTR 3:invNorm(0.995))

(b) *95% CI, proportion of student heights over 6 feet tall.*
37 of 102 students, chosen at random from 4000 PNC, over 6 feet tall.

i. *Point estimate*
Point estimate of proportion, p, of student heights over 6 feet tall is $\hat{p} = \frac{37}{102} \approx$ (choose one) **0.363** / **0.378** / **0.391**.

ii. *Check assumptions.*
Since $n\hat{p}(1-\hat{p}) = 102\left(\frac{37}{102}\right)\left(1 - \frac{37}{102}\right) \approx 23.6 > 10$, and $n = 102 < 0.05(4000) = 200$, assumptions (choose one) **have** / **have not** been satisfied and so it is appropriate $\hat{p} \pm z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ estimate parameter p.

iii. *95% CI of population proportion, p, of PNC students over 6 feet tall.*
$\hat{p} \pm z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \frac{37}{102} \pm 1.96 \times \sqrt{\frac{\frac{37}{102} \times \frac{65}{102}}{102}} =$ (choose one) **(0.269, 0.456)** / **(0.273, 0.367)** / **(0.233, 0.367)**.
(STAT TESTS 1–PropZInt... 37 102 0.95 Calculate)

3. *Sample size given margin of error and level of confidence.*
Sample size necessary to achieve a required margin of error, E, with a given level of confidence in a confidence interval determined using formula, if *prior* $\hat{p}$ available,

$$n = \hat{p}(1-\hat{p})\left(\frac{z_{\frac{\alpha}{2}}}{E}\right)^2,$$

and if prior $\hat{p}$ unavailable,

$$n = \frac{1}{4}\left(\frac{z_{\frac{\alpha}{2}}}{E}\right)^2.$$

(a) *Sample size for proportion p with prior $\hat{p}$: purchase slips.*
In an initial simple random sample, twenty–five (25) of 100 purchase slips chosen are Visa. What is sample size, n, required to estimate proportion Visa purchase slips, p, to within margin of error of $E = 0.01$ with 85% confidence? Here

$$n = p(1-p)\left(\frac{z_{\frac{\alpha}{2}}}{E}\right)^2 = \left(\frac{25}{100}\right)\left(\frac{75}{100}\right)\left(\frac{1.44}{0.01}\right)^2 \approx$$

(circle one) **3888** / **5184** / **5470**.

(Prior $\hat{p} = \frac{25}{100}$. Use invNorm(0.925) for $z_{\frac{\alpha}{2}} = z_{\frac{0.15}{2}} = z_{0.075} = 1.44$.)

(b) *Sample size for proportion p without prior $\hat{p}$: purchase slips.*
What is sample size, n, required to estimate proportion Visa purchase slips, p, to within margin of error of $E = 0.01$ with 85% confidence? Here

$$n = \frac{1}{4}\left(\frac{z_{\frac{\alpha}{2}}}{E}\right)^2 = \frac{1}{4}\left(\frac{1.44}{0.01}\right)^2 \approx$$

(circle one) **4409** / **5184** / **5470**.
Without prior $\hat{p} = 0.25$, sample size (choose one)
decreases / **remains same** / **increases** from $n \approx 3888$ to $n \approx 5184$.

(c) *Sample size for proportion p without $\hat{p}$: purchase slips.*
What is sample size, n, required to estimate proportion of Visa credit cards, p, to within margin of error of $E = 0.02$ with 90% confidence? Here

$$n = \frac{1}{4}\left(\frac{z_{\frac{\alpha}{2}}}{E}\right)^2 = \frac{1}{4}\left(\frac{1.65}{0.02}\right)^2 \approx$$

(circle one) **1702** / **1884** / **2470**.
(Use invNorm(0.95) for $z_{\frac{\alpha}{2}} = z_{\frac{0.10}{2}} = z_{0.05} \approx 1.65$.)

4. *Some comments*

(a) **True** / **False**

$$E = z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

(b) *Population, Sample, Statistic*[6] *and Parameter.* Match columns.

terms	credit card example
(a) population	**(a)** Visa or not, all purchase slips
(b) sample	**(b)** proportion of all slips made with Visa, p
(c) statistic	**(c)** Visa or not, 180 purchase slips
(d) parameter	**(d)** proportion of 180 slips made with Visa, $\hat{p}$

terms	(a)	(b)	(c)	(d)
credit card example				

9.4 Confidence Intervals for a Population Standard Deviation

After looking at chi–square distribution, we use it for $(1-\alpha)\cdot 100\%$ CI for σ^2:

$$\left(\frac{(n-1)s^2}{\chi^2_{\alpha/2}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}\right)$$

[6] Anything with a "hat" on it is a statistic; for example, $\hat{p}$.

and used when underlying distribution is normal with no outliers and sample is obtained using simple random sampling[7].

Exercise 9.4 (Confidence Intervals for a Population Standard Deviation)

1. *Probabilities for chi–square: waiting time to order*
   At McDonalds in Westville, waiting time to order (in minutes), X, follows a *chi–square*, χ^2, distribution. Consider following figure with two χ^2 distributions, each with different shaded areas (probabilities).

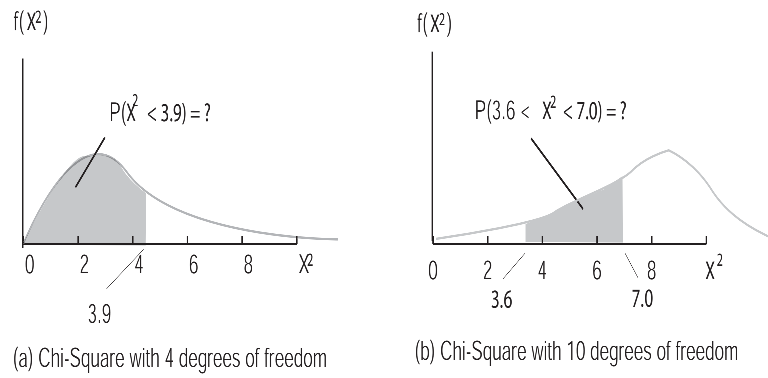

Figure 9.8 (Probabilities for χ^2 distribution.)

(a) For a χ^2 with 4 df, probability of waiting less than 3.9 minutes
$P(\chi^2 < 3.9) =$ (circle one) **0.35** / **0.45** / **0.58** / **0.66**
(Use 2nd DISTR χ^2cdf(0,3.9,4).)

(b) For a χ^2 with 10 df,
$P(3.6 < \chi^2 < 7.0) =$ (circle one) **0.24** / **0.34** / **0.42** / **0.56**.
(Use 2nd DISTR χ^2cdf(3.6,7.0,10).)

(c) Like t distribution, different χ^2 distributions indexed by degrees of freedom (df), which equal df $= n - 1$. For $n = 5$, df $= n - 1 = 5 - 1 =$ **3** / **4** / **5**.

(d) The χ^2 distribution with sample of size $n = 5$ (df $= 4$), in (a) of figure above, has mode (high point) at $n - 2 =$ (circle one) **1** / **2** / **3**.

(e) The χ^2 distribution is (choose one) **symmetric** / **asymmetric** but becomes more symmetric as df increase.

(f) Total area (probability) under chi-square is **50%** / **75%** / **100%** / **150%**.

(g) **True** / **False** Values of χ^2 are greater than or equal to zero.

[7]This CI for σ^2 can be converted to CI for σ by taking square-root. CI is sensitive to non-normal data which is not always fixed by large sample size. There is no preset menu in TI-84+ for this CI.

2. *Percentiles and critical values for chi–square: waiting time to order*
At McDonalds in Westville, waiting time to order (in minutes), X, follows a *chi–square*, χ^2, distribution. Consider following figure with two χ^2 distributions, each with 72nd percentile waiting time.

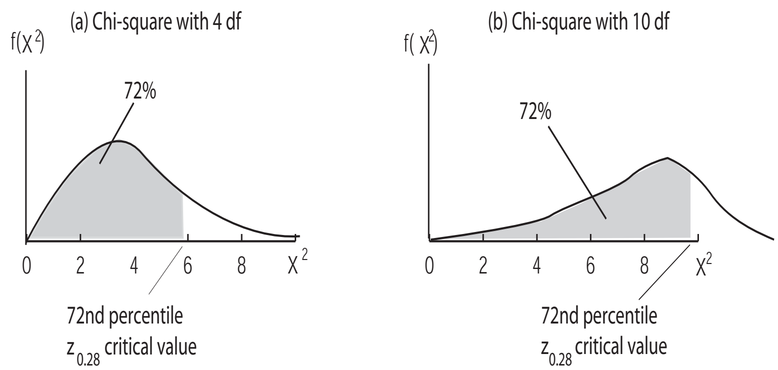

Figure 9.9 (Percentiles for χ^2 distribution.)

(a) The 72nd percentile waiting time for a χ^2 with 4 df, is (circle one) **3.1** / **5.1** / **8.3** / **9.1**.
(Use PRGM INVCHI2 ENTER 4 ENTER 0.72 ENTER)

(b) 72nd percentile or critical value $\chi^2_{0.28}$ waiting time with 10 df, is (circle one) **2.5** / **10.5** / **12.1** / **20.4**.
(Use PRGM INVCHI2 ENTER 10 ENTER 0.72 ENTER)

(c) The 32nd percentile or $\chi^2_{0.68}$ critical value for a χ^2 with 18 df, is (circle one) **2.5** / **10.5** / **14.7** / **20.4**.
(Use PRGM INVCHI2 ENTER 18 ENTER 0.32 ENTER)
The 32nd percentile is that waiting time such that 32% of waiting times are less than this waiting time and 68% are more than this time.

(d) If $\alpha = 0.20$ and $n = 19$,
$\chi^2_{\frac{\alpha}{2}} = \chi^2_{\frac{0.20}{2}} = \chi^2_{0.10} =$ (circle one) **20.5** / **21.5** / **24.7** / **26.0**.
(Use PRGM INVCHI2 ENTER 18 ENTER 0.90 ENTER)

(e) If $\alpha = 0.20$ and $n = 19$,
$\chi^2_{1-\frac{\alpha}{2}} = \chi^2_{1-\frac{0.20}{2}} = \chi^2_{0.90} =$ (circle one) **10.9** / **11.5** / **14.7** / **19.4**.
(Use PRGM INVCHI2 ENTER 18 ENTER 0.10 ENTER)

3. *Estimation for variance: car door and jamb.*
In a simple random sample of 28 cars, variance in gap between door and jamb is $s^2 = 0.7$ mm^2. Calculate 95% CI. Assume normality with no outliers.

(a) *Upper* critical value for $95\% = (1 - \alpha) \cdot 100\% = (1 - 0.05) \cdot 100\%$ CI is
$\chi^2_{\frac{\alpha}{2}} = \chi^2_{\frac{0.05}{2}} = \chi^2_{0.025} =$ (circle one) **8.7** / **40.1** / **43.2**
(Use PRGM INVCHI2 ENTER 27 ENTER 0.975 ENTER)

(b) *Lower* critical value for 95% = $(1-\alpha)\cdot 100\% = (1-0.05)\cdot 100\%$ CI is $\chi^2_{\frac{1-\alpha}{2}} = \chi^2_{1-\frac{0.05}{2}} = \chi^2_{0.975}$ = (circle one) **14.6** / **40.1** / **43.2**
(Use PRGM INVCHI2 ENTER 27 ENTER 0.025 ENTER)

(c) So, 95% CI is

$$\left(\frac{(n-1)s^2}{\chi^2_{\alpha/2}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}\right) = \left(\frac{(28-1)0.7}{43.2}, \frac{(28-1)0.7}{14.6}\right) =$$

(circle one) $\mathbf{(0.61, 1.65)}$ / $\mathbf{(0.59, 1.29)}$ / $\mathbf{(0.43, 1.29)}$.

(d) Since 95% CI (0.43, 1.29) does *not* include 0.40, this *indicates*[8] variance in distance between door and jamb (circle one) **is** / **is not** 0.4 mm^2.

(e) *Population, parameter, sample and statistic.* Match columns.

terms	jamb example
(a) population	**(a)** variance in jamb–door distance, of 28 cars, s^2
(b) sample	**(b)** variance in jamb–door distance, of all cars, σ^2
(c) statistic	**(c)** jamb–door distances, of all cars
(d) parameter	**(d)** jamb–door distances, of 28 cars

terms	(a)	(b)	(c)	(d)
jamb example				

4. *Estimation for variance: machine parts.*
In a simple random sample of 18 machine parts, variance in lengths is $s^2 = 12^2$. Calculate 90% CI. Assume normality with no outliers.

(a) *Upper* critical value for 90% = $(1-\alpha)\cdot 100\% = (1-0.10)\cdot 100\%$ CI is $\chi^2_{\frac{\alpha}{2}} = \chi^2_{\frac{0.10}{2}} = \chi^2_{0.05}$ = (circle one) **8.7** / **27.6** / **43.2**
(Use PRGM INVCHI2 ENTER 17 ENTER 0.95 ENTER)

(b) *Lower* critical value for 90% = $(1-\alpha)\cdot 100\% = (1-0.10)\cdot 100\%$ CI is $\chi^2_{\frac{1-\alpha}{2}} = \chi^2_{1-\frac{0.10}{2}} = \chi^2_{0.95}$ = (circle one) **8.7** / **40.1** / **43.2**
(Use PRGM INVCHI2 ENTER 17 ENTER 0.05 ENTER)

(c) So, 90% CI is
$\left(\frac{(n-1)s^2}{\chi^2_U}, \frac{(n-1)s^2}{\chi^2_L}\right) = \left(\frac{(18-1)12^2}{27.6}, \frac{(18-1)12^2}{8.7}\right) =$
(circle one) $\mathbf{(80.5, 101.4)}$ / $\mathbf{(100.5, 104.2)}$ / $\mathbf{(88.7, 281.4)}$.

(d) Since 95% CI (88.7, 281.4) includes test statistic $13^2 = 169$, this *indicates* variance in lengths (circle one) **is** / **is not** $\sigma^2 = 13^2$ mm^2.

[8]Population variance may be between 0.43 and 1.29, but we are 95% *confident* it is not.

9.5 Putting it Together: Which Procedure Do I Use?

As will soon be discovered (or already has been discovered), the main difficulty with confidence intervals is not so much to do with calculating them, as much as to do with deciding which one to calculate. Following table summaries confidence intervals given in this chapter and under what circumstances to calculate any one of these confidence intervals; other confidence intervals are given in later chapters. A table similar to this one will be used in later chapters to summarize tests of hypotheses.

CONFIDENCE INTERVALS		mean μ	variance σ^2	proportion p
	one	known σ: $\bar{x} \pm z_{\frac{\alpha}{2}} \left(\frac{\sigma}{\sqrt{n}}\right)$ unknown σ: $\bar{x} \pm t_{\frac{\alpha}{2}} \left(\frac{s}{\sqrt{n}}\right)$	$\left(\frac{(n-1)s^2}{\chi^2_{\alpha/2}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}\right)$	$\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
sample	two	chapter 11	chapter 11	chapter 11
	multiple	not covered for confidence intervals	not covered for CIs	not covered for CIs