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Chapter 5 Integers

Where in real life do you use or need negative numbers?

Temperature
Balancing the checkbook
Sea level
A loss of yards in a football game
A golf score
Betting on athletics games
When the VCR rewinds

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This discussion addresses not only how children learn but how integers and oepartions on integers might be taught to children.

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Should teachers ever teach the rules for integer operations without explaining them?

Is explaining the rules to children the same thing as children understanding the rules?

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5.1 Addition and Subtraction of Integers

A number line is a great tool to illustrate addition and subtraction of positive and negative numbers.

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Absolute value is important mathematically but not typically taught in elementary school.

5.2 Multiplication and Division of Integers

Some students may give rules for multiplication and division of positive and negative numbers but not reasons. Preservice teachers may need to explain or demonstrate why these rules work.

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5.3 Properties of Integers

Mathematics is inherently aesthetically pleasing. The key is to help students see some of the beauty of mathematics.

The actual names of the properties are not as important as the intuitive understanding of the properties!

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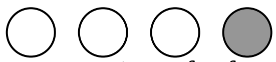
Chapter 6 Rational Numbers – Fractions

6.1 Fractions

Draw a circle, divide it into 4 equal sections, and then shade in one section. Ask the class what this represents.

This is a typical representation of a fraction of a whole or a continuous region.

Then draw four circles, shade in one of them, and ask students what this represents.



This is a typical representation of a fraction of a set or a discrete set of objects.

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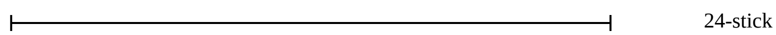
Why would a teacher not teach fraction as a location on a number line to children first learning about fractions?

Why is 1 /2 the same thing as 1 ÷ 2?

Fraction Concepts

Problems using segmented “sticks” help children link their part/whole view of fractions with their multiplicative understanding and thus help them draw on their natural-number knowledge to construct ideas about fractions. The teacher can introduce and “name” the 1- through 10-sticks shown below.

Then the teacher explains that she will make a stick by joining copies of a single stick from the set of 1- through 10-sticks. For example, the teacher may draw the following stick and pose the question, “The stick that I used was one-third of the length of the stick I have here. What stick did I use?”



A student may reason, “You used one-third, so 3 times what is 24? I think you used the 8-stick because 3 times 8 is 24.” The student may draw three copies of the 8-stick or iterate 8 three times (i.e., 8, 16, 24) to check. Thus the student conceives of one-third as one out of three equal parts in the whole and connects this view with his multiplicative understanding of 24 as composed of three 8s (Olive, 2002). This type of activity also helps children begin to view unit fractions such as one-third as iterable units. In other words, they see two-thirds as two one-thirds, three-thirds or the whole as three one-thirds, four-thirds as four one-thirds, and so on.

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Another example of how questions of this sort allow children to capitalize on their natural-number knowledge when thinking about fractions is illustrated in the following interview excerpt (Sáenz-Ludlow, 1994):

> T: If I give you forty-fiftieths of 1000 dollars, how much money will I give you?
> S: [After some thinking] Eight hundred dollars.
> T: Why?
> S: [Quickly] Because one-fiftieth is 20 dollars and five 20s is 100, so five, ten, fifteen, twenty, twenty-five, thirty, thirty-five, forty [keeping track of the counting of fives with his fingers and finally showing eight fingers]; that is 800.

In other words, once he determined that one-fiftieth was 20 dollars, the child reasoned that five-fiftieths was 100 dollars and then counted by fives to count to forty-fiftieths. Since each group of five-fiftieths was 100 dollars, the eight fingers he had showing after counting to forty-fiftieths indicated 800 dollars. Thus, this question allowed the child to count fractional units by counting natural-number units. By considering 100 as a composite unit made up of five twenties and coordinating these five twenties with five-fiftieths, the child was able to determine the value of forty-fiftieths by counting by fives and, implicitly, by one hundreds.

Note: Although the money tasks described above use well-specified wholes (e.g., 50 cents, two dollars, 1000 dollars) in order to help children build their fraction concepts from their knowledge of natural numbers, it is important to move on to tasks using nonspecific wholes (e.g., a "certain amount" of money) so that children may develop more generalizable fraction concepts that are not constrained by their knowledge of natural numbers.

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If possible assign the fraction interview to your students. They may be able to interview an adult who does not have a good understanding of fractions.

If a student got 2 out of 3 right (2/3) on the first quiz and 3 out of 4 right (3/4) on the second quiz, then they got 5 out of 7 right (5/7) which is not 2/3 + _ ≠ 5/7?

This example also relates to question #9 of the fraction interview?

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Children first come to understand fraction as probably a fraction of a whole and then maybe as a fraction of a set even though fraction has other interpretations.

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Fraction Bars

After group work, discuss the solutions. Make a list of the different solutions on the board or overhead.

Returning to the list of fractional number sentences equal to 1, pick out an example and compute the sum by finding a common denominator. Pick an example like the following:

$$2/5 + 1/3 + 1/4 = 1$$

$$\begin{array}{rr} & 2/5 = 48/120 \\ & 1/3 = 40/120 \\ + & 1/4 = 30/120 \\ \hline & 118/120 \end{array}$$

Compare this understanding of fractions with the student who said that 3/4 + 3/4 is less than one. Here, 118/120 is within 2/120 (1/60) of a whole, which would be a very small fold in the strips.

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6.2 Addition and Subtraction of Fractions

Illustrate that 1/3 + 1/2 is 5/6 symbolically by finding a common denominator and solving.

What do you get when you add 1/2 of an apple and 1/2 of an orange? **Fruit salad!**

Adding and subtracting fractions is more difficult conceptually than multiplying and dividing fractions.

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Fraction Bars are a good manipulative to use to demonstrate addition and subtraction of fractions.

Multiplication and Division of Fractions

Intuitively children understand that multiplication makes bigger and division makes smaller. It is okay for teachers to emphasize this when children are learning multiplication and division of whole numbers because this is how children think children think about these operations!

Preservice teachers should be able to demonstrate (not prove) multiplication and division of fractions to children, even though the rules, in isolation, may appear “simple.” Every attempt should be made to make mathematics a sense making activity (cf. 1.3). Research with older students shows that they often mix up the rules for working with fractions.

It is important to encourage prospective teachers to examine their own understanding of the standard algorithms for adding, subtracting, multiplying, and dividing fractions because, in the course of doing so, they will often realize that they do not fully comprehend why these procedures work. Asking them to respond to “why-does-that-work?” questions can also stimulate their interest in children’s ways of thinking (Tirosh, 2000). Some questions you might ask include:

1) Why do we need to get a common denominator when adding or subtracting fractions?
2) Why do we “invert and multiply” when dividing fractions?
3) Why do we multiply numerator times numerator and denominator times denominator when multiplying fractions.
4) When converting a mixed number to an improper fraction, why do we multiply the whole number times the denominator of the fractional part, add this product to the numerator of the fractional part, and write the result over the denominator of the fractional part?
5) Why does the strategy for comparing two fractions by cross-multiplying work?

Another good activity is to ask prospective teachers to think of examples of daily situations that involve addition/subtraction of fractions, multiplication of fractions, and division of fractions. For example, can they think of “real-life” problems that could be represented by number sentences such as $\frac{1}{2}+\frac{1}{3}$, $\frac{1}{2}\times\frac{1}{3}$, $\frac{1}{2}\div\frac{1}{3}$? When thinking about division situations, ask them to reflect on what model of division they are using.

The following two activities, taken from Tirosh (2000), may also promote good class discussions:

Activity 1: Can Fractions Be Computed in an Easier Way?

1. You are discussing operations with fractions in your class. During this discussion, Ron says

   It is easy to multiply fractions; you just multiply the numerators and the denominators. I think that we should define the other operations on fractions in a similar way:

Addition	$\frac{a}{b}+\frac{c}{d}=\frac{a+c}{b+d}$
Subtraction	$\frac{a}{b}-\frac{c}{d}=\frac{a-c}{b-d}$
Division	$\frac{a}{b}\div\frac{c}{d}=\frac{a\div c}{b\div d}$

How will you respond to Ron's suggestions? (Deal with each separately.)

2. Shereka agrees with Ron and adds, "I don't understand why they decided to define addition, subtraction, and division in such complicated ways. Why didn't they decide to define the operations in the way that Ron suggested?" How will you respond to Shereka's question?

Activity 2

1. How would you explain to someone why $\frac{2}{3}\div\frac{1}{3}=2$? Why $\frac{2}{3}\div\frac{1}{6}=4$?
2. Derek argues that he prefers to divide fractions in a way similar to multiplication. For instance, $\frac{2}{9}\div\frac{1}{3}=\frac{2\div 1}{9\div 3}=\frac{2}{3}$. Would you accept Derek's proposal? Why?
3. We have discussed two interpretations of division: sharing and measurement. An example of a sharing division word problem is

   Mother divided 6 candies evenly among her 3 children. How many candies did each child receive?

   An example of a measurement division word problem is

   Six candies were divided among some children. Each child got two candies. How many children received candies?

   a. Write sharing and measurement division word problems, using fractions instead of natural numbers, and discuss the problems you faced when doing so.
   b. What constraints does each of these models impose?

4. Write expressions that solve the following word problems. List possible difficulties you think children may commonly have when solving these problems and describe the possible sources of the difficulties.
   a. An 8-meter-long stick was divided into 13 equal pieces. What was the length of each piece?
   b. Six kilograms of cheese were packed in boxes, each box containing $\frac{3}{4}$ of a kilogram. How many boxes were needed to pack all the cheese?

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Fractions with 0 should can be related to division by and with 0.

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6.4 Properties of Rational Numbers

Typically, the denseness property is not taught directly to children, but a common activity is to have children find a fraction between two other fractions. Hopefully, preservice teachers will see the intent of such activities as finding a fraction between 1/5 and 1/6for children.

The closure property drives our creation of rational numbers. Again this illustrates that numbers are created out of need and not just arbitrarily.

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